“Assessing and Managing Operational Risk with a Special Emphasis on Terrorism Risk”

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Preface

I begin by thanking my supervisor Prof. Marc Chesney for inspiring me to do my research in the area of terrorism risk and its impact on the behavior of financial markets. I would also like to express my sincere appreciation of his kind guidance through the course of my research, the numerous discussions and considerable amount of advice that, all together, made it possible to develop my research. I am also very grateful to the co-referee Prof. Rajna Gibson for her valuable recommendations and helpful comments on my work.

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Zurich, April 2, 2008 Ganna Reshetar

Contents

Introduction

The objective of this thesis is to consider different risk management issues in relation to operational risk with a special emphasis on terrorism risk. Our motivation to implement research in this particularly challenging area of risk management is due to the increasing frequency and magnitude of operational losses over the last decade and their negative effect on financial industry[1]. This thesis contributes to the existing research on operational risk in several ways. First, our research suggests a model that addresses the issue of dependence between operational losses and how it can be accounted for in the value of capital charge for operational risk. Second, we provide a better understanding of the impact of a particular type of operational risk event, specifically of terrorist attacks. As evidenced by the 9/11 attacks, this risk can be catastrophic and can have negative consequences on the behavior of financial markets. We implement empirical analysis of the impact of terrorist attacks on stock, bond and commodity markets and suggest possible diversification strategies of terrorism risk. Finally, we contribute to the area of operational risk transfer, by developing a model for pricing of a multiple-event coupon paying CAT bond. The bond that we consider covers exposure to catastrophic risk such as natural and man-made disasters, including terrorist events.

Market and credit risks have been the subject of much debate and research in the financial industry during the 1990s. Since then financial institutions have made significant

[1] Rising levels of exposure to operational risk are driven by deregulation, globalization, and advances in technology, that in turn, have led to the creation of new and highly sophisticated production processes and complex products. For example, the development of hedging and risk mitigation techniques have enabled financial institutions to better manage the market and credit risks arising from complex products but, in turn, have created additional operational risk exposures. Another example, the growth of e-banking and e-commerce that exposes institutions to such type of operational risk as fraud. The upward trend in operational loss frequencies and severities is described in papers by Cummins, Lewis, and Wei (2006) and Fontnouvelle, De Jesus-Rueff, Jordan, and Rosengren (2006). Finally, terrorist attacks have brought increased attention to operational losses. The magnitude and frequency of these operational risk events have been increasing since 1982 (OECD (2005)).

progress in the identification, measurement and management of both types of risk. However, experience of events such as the 1995 bankruptcy of Barings Bank due to fraudulent trading (Brown and Steenbeek (2001)), the losses in financial industry due to the 9/11 attacks (Johnston and Nedelescu (2005)) and losses due to data input error in Mizuho Financial Group in 2005 (Katsumura and Obayashi (2005)) highlights the fact that risk management should go beyond market and credit risks only. We are referring to operational risk: the risk of an institution experiencing losses due to an internal system failure or due to external events, such as terrorist attacks and natural disasters. Initially, this risk was defined as any form of risk that is not market or credit risk. However, this definition is rather vague and does not tell us about the exact types of operational risk faced by financial institutions. A better definition is provided by the regulators, who define operational risk as "*...the risk of direct or indirect loss resulting from inadequate or failed internal processes, people and systems or from external events*" (BIS (2006)). This definition includes legal risk, but excludes strategic and reputational risk. Together with researchers and risk managers at major banks, the Basel Committee on Banking Supervision has developed a risk management framework for operational risk that includes qualitative and quantitative techniques to assess potential exposure to this risk as well as approaches to compute a capital charge against operational losses (BIS (2003a), BIS (2003b) BIS (2005), BIS (2006)). In addition, the literature on the conceptual issues and quantitative methodologies in relation to this risk both by scientific researchers and practitioners has been developing (see Frachot, O.Moudoulaud, and T.Roncalli (2003), Fontnouvelle and E.Rosengren (2004), Moscadelli (2004), Fontnouvelle, V.DeJesus-Rueff, J.Jordan, and E.Rosengren (2003), Alvarez (2001), Embrechts, C.Kluppelberg, and T.Mikosch (1997), McNeil and Saladin (1997) and McNeil (1999)). In this thesis we contribute to this research area by looking at different aspects of operational risk management.

First, we suggest a model that addresses the issue of dependence between operational losses and how it can be accounted for in the value of capital at risk (CaR) associated with operational risk. In contrast to the Loss Distribution Approach (LDA) described by regulators in Basel II, our model accounts for the underlying dependence between aggregate losses of different classes of risk. For the first time in this research area, we model this dependence, assuming that it is driven by *both*, dependence between loss frequencies and dependence between loss severities. The model implementation shows that values of capital at risk obtained based on the loss distribution approach are higher than values obtained using the suggested approach. In addition, we demonstrate results of the numerical evaluation when instead of the VaR, a coherent and more conservative risk measure is used to compute a capital charge, namely the expected shortfall (ES). This measure reflects a possible exposure to a loss that exceeds VaR. Given possibility of rare but high magnitude operational risk 'tail events', the ES is more reliable than VaR.

The findings of the model reveal that accounting for operational loss dependence in the value of capital at risk improves its accuracy. This is important for banks and regulators because for banks, the capital charge should reflect their true operational risk exposure. For regulators, the capital charge that is correctly computed across all banks would help to preserve stability in the banking and financial sectors of the economy.

Second, we implement empirical analysis of the impact of operational risk events such as terrorist attacks on the behavior of different financial markets. The impact on financial markets of the 9/11 terrorist attacks as well as those of more recent attacks in Madrid in 2004 and London in 2005 reveals that terrorism risk is a new type of operational risk that can be catastrophic and that investors and financial institutions are likely to face in the future.

In our study, we consider terrorist events that took place in 25 countries over an 11 year time period and implement our analysis using different methods: an event-study approach, non-parametric methodology, and a filtered GARCH-EVT approach. The results of this study show that although financial markets perceive terrorist events as unusual, they do not see their effects as long-lasting. The results of empirical analysis suggest several diversification strategies for investors who may be concerned about possible adverse effects of terrorism risk on their portfolios. When dealing with terrorism risk, investors should hold assets that are likely to react positively to terrorist attacks or, those that have little or no negative sensitivity to this risk. The first type of asset may be represented by a U.S. Government bond index followed by such industry stocks as aero/defense and pharma/biotech. The second type of assets may include a banking stock index (unless an event similar to the 9/11 attacks occurs in the heart of financial industry). Note that, though this stock index is least sensitive to terrorist attacks, it exhibits significant negative return movements associated with financial crashes. To reduce negative exposure to terrorist events investors should avoid investing in insurance, travel and airline industry stocks. As to commodities, investing in a composite commodity index is preferable to investing in gold only.

The response of financial markets to terrorist events suggests several strategies of trading derivatives. For example, investors can hold long positions in put options on industry stocks that react negatively to terrorist events (for example, airline and insurance industry stocks). Or alternatively, they can invest in call options, where the underlying asset is a U.S. Government bond index.

Finally, we find both similarities and differences between the impact of terrorist events on financial markets and the effect of financial crashes and natural disasters. Note that the recent history of natural disasters and terrorist events reveals a changing nature of these catastrophic events both in terms of their frequency and their magnitude. The anticipated increase of severe storms and weather events associated with climate change and on-going threat of terrorist events can place enormous financial demands on the insurance and reinsurance businesses (Fishel (2005)). As a result, development of the ways to transfer these operational/catastrophic risks to capital markets has become more important than ever before. The final contribution of our thesis addresses this very issue. We develop a framework for pricing of a multiple-event coupon paying CAT bond. The model is the first of its kind to address theoretical issues of pricing of an insurance-linked security that derives its value based on two underlying processes: catastrophic insured property losses and catastrophic mortality. It is also the first study that develops a CAT bond with a multiple-event and multi-risk structure. In addition, this work provides a numerical evaluation of the bond's price using the UK catastrophic data provided by Swiss Re. The results of this study indicate that the price of the bond increases with attachment levels and decreases with stronger positive dependence between property losses and deaths. Our research finds presence of an inverse relationship between the price of the bond and its time to expiration. Although this relationship always works for a zero-coupon catastrophe bond, it may not always hold for those cases where the bond pays coupons. We find a less responsive behavior of the bond's price to changes in dependence compared to changes in the time to maturity. Overall, the model suggested in this paper may be interesting to insurance and reinsurance companies and other financial institutions that want to transfer their exposure to catastrophic risks, including risk of terrorism, to capital markets. With respect to the latter risk, our model allows to protect from losses and deaths that are of significantly lower magnitude than what current terrorism related risk transfer securities allow.

In summary, the thesis consists of three papers that focus on different aspects of operational risk management. Paper 1 addresses the issue of dependence between operational losses and how it can be accounted for in the value of capital at risk of a bank. Paper 2 describes empirical analysis of the impact of terrorism as a particular type of operational risk on the behavior of stock, bond and commodity markets. This paper is a joint work with Prof. Dr. Marc Chesney. Paper 3 presents a framework for pricing of a multiple-event coupon paying catastrophe risk bond. This type of a bond represents one of the ways to transfer to capital markets operational risk related to terrorism and natural catastrophes.

Paper 1

Dependence of Operational Losses and the Capital at Risk

1.1 Introduction

Since the time of famous Barings fraudulent trading in 1995 (Bhugaloo (2005)), operational risk has become an area of growing concern in banking. The magnitude of operational losses over the last decade and their negative effect on banks' financial position and reputation explains increasing attention to the management of operational risk. The New Basel Capital Accord (Basel II) proposed by the Basel Committee on Banking Supervision calls for an explicit treatment of operational risk and asks banks to set a capital charge against operational losses (BIS (2003a), BIS (2003b) BIS (2005), BIS (2006)). This paper addresses the issue of dependence of operational losses and how it can be accounted for in the value of capital at risk (CaR) of a bank. In contrast to the Loss Distribution Approach (LDA) described by regulators in Basel II, our model accounts for underlying dependence between aggregate losses of different classes of risk. For the first time in this research area, we model this dependence, assuming that it is driven by *both* dependence between loss frequencies and dependence between loss severities. To consider the latter type of dependence is as important as to consider the frequency dependence since there may be an external event that effects not only occurrence of losses but also their sizes simultaneously in different departments of a bank[1]. Though severity dependence has been suggested in some papers (see Lindskog and A.McNeil (2003), Chapelle, Y.Crama, G.Hubner, and J.Peters (2004)), it has never been computationally incorporated into existing models, leaving this issue for future research. Our approach helps to decrease inaccuracy in the measurement of bank's exposure to operational risk by accounting for these two sources of dependence.

When implementing our computations for different cases of loss dependence, we examine the difference in values of the CaR obtained using the model suggested and via the LDA. The underlying assumption of the LDA is rather conservative: the capital charge is computed as a simple sum of the operational risk VaR for each class of risk (business line/risk type cell)[2]. Frachot, T.Roncalli, and E.Salomon (2004) show that this way of computing CaR is equivalent to assuming perfect correlation across operational losses of different classes of risk. This means that in case of an operational risk event, the latter would cause operational losses simultaneously in all divisions of a bank. Though this may happen, for example, in case of an event similar to the 9/11, this is not likely in general. Therefore if the underlying loss correlation is not perfect, the LDA would result in overestimation of aggregate losses and the CaR value.

The results of this work show that in all cases considered, values of the CaR obtained based on the LDA approach are higher than values obtained via the model suggested. This finding supports the hypothesis about overestimation of the CaR computed via the regulatory LDA unless the underlying loss dependence is perfect. Another important result, which is in line with this hypothesis, shows that the closer loss dependence between risk classes approaches one, the smaller the difference between CaRs computed in these two models. Implementation of the model results in the following reduction of the CaR when compared to the corresponding values obtained based on the LDA: 18.51% in the

[1]It is a common argument in the operational risk modelling literature that operational losses do in general exhibit dependence in their severity (see Chavez-Demoulin, Embrechts, and Neslehova (2006), Lindskog and A.McNeil (2003)). Think about major economic events, weather catastrophes or terrorist attacks like September 11. Such severe events will typically effect several business lines and cause simultaneous increase in operational loss sizes across them. For example, banks in Manhattan have experienced increase in the loss amounts in trading and sales as well as in payment and settlement due to the 9/11 attacks (see Lacker (2003), Bonturi, Koen, and Lenain (2002), Johnston and Nedelescu (2005), Chen and Siembs (2004), Kaganoff-Stern (2004)).

[2]According to BCBS, each class of risk of a bank includes losses within a certain business line as for example, BL "Payment and Settlement", and that are due to a certain operational risk type as for example, "Internal Fraud". Basel II defines 7 risk types and 8 business lines, which results in 56 risk classes.

case of aggregate loss independence and 2.78% in the case when loss dependence measured by the Kendall's tau is 0.78.

In addition, we implement numerical evaluation of the capital charge when it is computed as expected shortfall (ES). This risk measure is not suggested by regulators, however its coherence property (see Artzner, F.Delbaen, Eber, and D.Heath (1999)) has advantageous implications on the capital at risk. ES is more conservative than VaR, which results in higher values of capital charge compared to the regulatory setting. At the same time, this measure reflects a possible exposure to extreme losses better: it tells about the potential size of the loss that exceeds VaR. Given possibility of rare but high magnitude operational risk 'tail events', the ES is more reliable than VaR. Our numerical analysis revels a similar level of capital reduction of the CaR computed as ES if compared to VaR: 18.05% in the case of aggregate loss independence and 3.39% in the case when loss dependence is 0.78.

Note that when computed by a bank, the reductions of the CaR can be higher or lower then those obtained in our numerical exercise. This is because parameters/types of the loss and frequency distributions of a class of risk of a bank can differ from those used in this study. In addition, the accuracy of the estimated results is negatively affected by a small sample size of simulated values of CaR (200 values). The latter is due to computational intensity of sampling dependent aggregate loss data and estimating $VaR_{99.9\%}$ and $ES_{99.9\%}$. Finally, the results obtained relate to the dependence between two classes of risk only, implying that for a bank, the reduction can be much higher when dependence between all 56 risk classes is considered.

The results of this paper show that accounting for loss dependence in the value of CaR improves its accuracy. This is important for banks and regulators because for banks, the capital charge should reflect their true operational risk exposure. For regulators, the capital charge that is correctly computed across all banks would help to preserve stability in the banking and financial sectors of the economy.

1.2 Related Research

Since operational risk management is a relatively new area in banking, the literature on this topic both by scientific researchers and practitioners, is currently growing, with a particular focus on quantitative methodologies and their implementation. One group of papers in this field describes practical implementation of the standard LDA described by BIS (2006). Papers by Frachot and T.Roncalli (2001), Frachot, O.Moudoulaud, and T.Roncalli (2003), Frachot, T.Roncalli, and N.Baud (2002) explore this approach for computing bank's capital charge for operational risk and show the way this capital can be allocated within different divisions of a bank.

To our knowledge, the first paper that addresses the issue of possible overestimation of the CaR for operational losses obtained via the LDA is the paper by Frachot, T.Roncalli, and E.Salomon (2004). The authors refer to the 'correlation problem'[3]. They show that following the LDA and computing the capital charge of a bank by summing up the capital charges across different classes of risk implies the assumption of perfect positive correlation between aggregate operational losses. They state that, with a strong or even perfect frequency correlation, the loss dependence on an aggregate level is lower and is not perfect. They propose a formula to compute CaR that accounts for underlying correlation between loss frequencies. Important limitation of their approach is that it ignores possible dependence between loss severities. As we mentioned before, it is as important to consider the severity dependence as it is to consider the frequency dependence. There are two papers, one by Lindskog and A.McNeil (2003) and another one by Chapelle, Y.Crama, G.Hubner, and J.Peters (2004) that recommend to consider severity dependence in the loss dependence modelling. However, in both papers, authors do not incorporate this dependence into the models they propose, leaving this issue for the future research.

As the standard LDA originates from actuarial techniques developed and used in the insurance industry, a lot of information on the subject can be found in books and papers by Klugman, H.Panjer, and G.Willmot (1998), Panjer (1981), Panjer and G.Willmot (1986), Robertson (1992), Venter (1983), Willmot and X.Lin (2000), Tripp, H.Bradley,

[3]Note that we prefer to refer to 'correlation problem' as 'dependence problem'. This is because for non-elliptical distributions, which is often true for loss distributions, correlation is not appropriate measure of dependence.

and R.Devitt (2004). When applying these techniques to operational loss data, it is important to account for its threshold and selection biases, poor quality and a small sample size. Baud, A.Frachot, and T.Roncalli (2002), for example, describe MLE that accounts for thresholds and therefore handles possible heterogeneity of data.

Quantification via the LDA assumes separate modelling of operational loss frequencies and operational loss severities. Research results in this area have shown that calibration of these distributions (i.e. the choice of the distribution type and estimation of its parameters) is the most demanding task because of the poor quality and a small sample size of available operational loss data (see Fontnouvelle and E.Rosengren (2004), Moscadelli (2004), Fontnouvelle, V.DeJesus-Rueff, J.Jordan, and E.Rosengren (2003), Alvarez (2001), Embrechts, C.Kluppelberg, and T.Mikosch (1997), McNeil and Saladin (1997), McNeil (1999), Chapelle, Y.Crama, G.Hubner, and J.Peters (2004) and Roehr (2002)).

Finally, there is an extensive literature on dependence modelling both, using copulas and mixture models. The latter models have been often described in the literature on credit risk (see, for example, Frey and McNeil (2003), Duffie and K.Singleton (1999) and Lando (1998)). As to copulas, we refer to the books by McNeil, Frey, and Embrechts (2005) and Cherubini, E.Luciano, and W.Vecchiato (2004) and papers by Embrechts, F.Lindskog, and A.McNeil (2001), Bouye (2000) and Dowd (2005) as useful sources on the subject.

1.3 The LDA for Operational Risk

Once considered primarily in the context of back-office functions, operational risk is relevant to almost all aspects of banking business. There is still ongoing debate concerning general definition of operational risk. Many institutions describe this risk as all risks other than market and credit risks. Regulators define this risk as "*...the risk of direct or indirect loss resulting from inadequate or failed internal processes, people and systems or from external events*" BIS (2006). Strategic and reputational risks are not included in this definition.

The regulatory capital scheme is based on three different methods: the Basic Indicator Approach (BIA), the Standardized Approach (SA) and the most sophisticated one, the Advanced Measurement Approach (AMA). Under the AMA, which is the most risk-sensitive, banks may choose the LDA (see Figure 1 in Appendix B) or alternatively, any other internal operational risk measurement model developed by a bank.

In the Basel framework, all operational losses are attributed to a "class of risk",where each class designates one cell among the 7 risk types $\times$ 8 business lines cells[4]. Implementation of the LDA involves five steps. First, banks model the severity of loss events. The amount of one loss event for the business line i and the event type j is a random variable $X(i,j)$. Within each class of risk, the loss amounts are independent and identically distributed. The distribution of loss amounts $F_{i,j}$ describes the currency values of bank's individual loss events.

The second step involves modelling of frequency of loss events. The number of losses $N(i,j)$ is random over a specified time horizon τ and frequency probability function $p_{i,j}$ provides information about the number of loss events that could occur between times t and $t+\tau$ and associated probabilities.

Estimation of the compound operational loss distribution $G_{i,j}$ of each class of risk is implemented in the third step. Based on the previously generated frequency and severity distributions it is defined as

$$G_{i,j}(x) = \begin{cases} \sum_{n=1}^{\infty} p_{i,j}(n) F_{i,j}^{n*} & \text{if } x > 0; \\ p_{i,j}(0) & \text{if } x = 0, \end{cases}$$

where $*$ is the convolution operator on distribution functions and F^{n*} is the n-fold convolution of F with itself. In general, there is no analytical expression of the compound loss distribution. Computing this distribution requires numerical algorithms such as the Monte Carlo method (Frachot and T.Roncalli (2001)), Panjer's recursive algorithm (Panjer (1981)), the Heckman-Meyers method (Heckman and G.Meyers (1983)) and the Fast

[4]The eight business lines are: Corporate Finance; Trading and Sales; Retail Banking; Payment and Settlement; Agency Services; Commercial Banking; Asset Management; and Retail Brokerage. The seven loss types are: Internal Fraud; External Fraud; Employment Practices and Workplace Safety; Clients, Products and Business Practices; Damage to Physical Assets; Business Disruption and System Failure; and Execution, Delivery and Process Management

Fourier Transform techniques (Robertson (1992), Wang (1998)).

Once estimated, banks use the compound loss distribution to obtain the CaR for each class of risk. BIS (2006) defines the regulatory capital charge (or the CaR) for operational risk as a Value-at-Risk measure[5] computed for one year holding period and $\alpha = 99.9\%$ confidence interval:

$$\mathrm{CaR}(i, j; \alpha) = G_{i,j}^{-1}(\alpha). \tag{1.1}$$

Finally, in the last step, the total capital charge for operational risk $\mathrm{CaR}(\alpha)$ of a bank is obtained by summing up capital charges across all classes of risk:

$$\mathrm{CaR}(\alpha) = \sum_{i=1}^{8} \sum_{j=1}^{7} \mathrm{CaR}(i, j; \alpha). \tag{1.2}$$

In their paper, Frachot, T.Roncalli, and E.Salomon (2004) show that computation of the total capital charge as it is presented in (2), implies the assumption of perfect positive dependence between aggregate losses of different classes of risk.

Note that the CaR computed as the VaR, has limitations as it is not a coherent risk measure in the sense of Artzner, F.Delbaen, Eber, and D.Heath (1999). It lacks the property of subadditivity and does not tell anything about the potential size of the loss that exceeds it. To circumvent this problem, the concept of expected shortfall has been introduced by the same authors. Basically, this measure is a conditional expectation of X, given that it exceeds a selected level L of the distribution:

$$\mathrm{ES}_\alpha(X) := E[X \mid X \geq L].$$

The level L can correspond to some threshold value or to be a value of the VaR itself. As coherent risk measure, expected shortfall is more advantageous than VaR. Consequently, we go beyond the regulatory setting and compute values of CaR not only as VaR, but also as values of expected shortfall for $L = VaR_\alpha$ and $\alpha = 99.9\%$:

$$\mathrm{ES}_\alpha(X) := E[X \mid X \geq \mathrm{VaR}_\alpha(X)].$$

In contrast to VaR, expected shortfall (ES) is not a point measure of risk - the shape

[5]Given a confidence level $\alpha \in (0, 1)$, VaR is equal to the smallest number x such that the probability that the loss X exceeds x is no greater than $1 - \alpha$: $\mathrm{VaR}_\alpha(X) := \inf\{x \in R \mid P(X > x) \leq 1 - \alpha\}$.

of the distribution enters into risk computations. As a result, banks have to require information about the tail of distribution of losses, which improves their risk management process.

1.4 The Model

1.4.1 Assumptions

As in the LDA, all operational losses are attributed to a class of risk. Each class of risk i covers losses within one business line and that are due to some risk event j. Between times t and $t+\tau$, the following assumptions are applied

- The amount (severity) of one loss event is a random variable $X_{i,j}$, $i = 1, ..., k$, $j = 1, ..., m$. The loss amounts $X_{i,j}$ are independent and identically distributed and follow severity distribution as $F_{i,j}(x)$. We assume that $F_{i,j}(x)$ is Pareto.

- The number of losses $N_{i,j}$ is random and has a probability function $p_{i,j}(n)$. The loss frequency distribution of $N_{i,j}$ is $P_{i,j}(n) = \sum_{q=0}^{n} p_{i,j}(q)$ and we assume that it is Negative Binomial.

- The common distribution of $X_{i,j}$ does not depend on $N_{i,j}$ and the distribution of $N_{i,j}$ does not depend on the values of $X_{i,j}$.

- For class of risk i with losses due to an event type j, the random sum $L_{i,j} = X_1 + \ldots + X_{N_{i,j}}$ corresponds to the aggregate loss amount and has the following distribution function

$$F_{L_{i,j}}(x) = \begin{cases} \sum_{n=1}^{\infty} p_{i,j}(n) F_{i,j}^{n*}(x) & \text{if } x > 0; \\ p(0) & \text{if } x = 0, \end{cases}$$

  where $*$ is the convolution operator on distribution functions and $F_{i,j}^{n*}$ is the n-fold convolution of $F_{i,j}$ with itself.

- There is dependence d_s between loss severities (amounts) of different classes of risk.

- There is dependence d_f between loss frequencies (occurrences) of different classes of risk.

- There is dependence d between *aggregate* losses of different classes of risk.

Since Pareto and Negative Binomial distributions belong to the class of non-elliptical distributions, we chose the Kendall's tau measure of dependence[6]. When implementing the model, we consider two classes of risk only and we assume that losses in each class of risk occur due to only one type of risk event.

1.4.2 Loss Dependence

We consider three *types* of loss dependence. The first type is dependence between loss severities. We may observe that historically, the loss amounts are high (or low) in one business line of a bank when the loss amounts are high (or low) in another business line of a bank. This can be explained by the fact that severities of losses in both business lines are dependent because they are driven by the same factor (macroeconomic, political, internal for a bank etc.). The second type of loss dependence, namely dependence between loss frequencies, can be explained in a similar way except that we consider the number of losses instead of loss amounts. Finally, after aggregation of losses on the level of a class of risk we obtained a third type of loss dependence, which is dependence between risk classes, and which is driven by the underlying frequency and severity dependencies.

We implement dependence modelling using mixture models. In general, these models have been commonly used in the literature on credit risk (for example, Duffie and K.Singleton (1999) and Lando (1998) use Bernoulli mixture models.). In this work, we use the common Exponential-Gamma mixture to model dependent severities and the common Poisson-Gamma mixture to model dependent frequencies. We chose these types of mixture models because they are easy to implement and allow us to model dependent Pareto severities and Negative Binomial frequencies of losses correspondingly. The choice of aforementioned distributions is justified by the results of the empirical studies by Moscadelli (2004) and Fontnouvelle, V.DeJesus-Rueff, J.Jordan, and E.Rosengren (2003).

[6] $d_{Kendall}(Y_a, Y_b) = \binom{n}{2}^{-1} \sum_{1 \le t < s \le n} sign((Y_{t,a} - Y_{s,a})(Y_{t,b} - Y_{s,b})$.

They show that loss severities attributed to some business lines of a bank are well modelled by Pareto distribution. As to loss frequencies, the studies mentioned above show that Negative Binomial distribution provides a very good fit to the occurrence of operational losses, and is a better choice than a Poisson distribution. The latter distribution implies a constant rate of loss occurrence over time, which is not the case in reality.

We assume that amounts of operational losses of different class of risk are due to realization of a particular random variable θ_s. This parameter is characterized by some distribution function. For any given θ_s, the individual conditional severity distributions are independent and follow exponential distribution. However, unconditionally, these distributions are dependent with dependence coming through the same random parameter θ_s. Similar to the severity case, there is an external factor θ_f that effects frequencies of operational losses. Given this parameter, individual conditional frequency distributions are independent and follow a Poisson process. Unconditionally, however, these distributions are dependent. By modelling the uncertainty about the external mechanism in both cases via the Gamma-distributed parameters θ_s and θ_f, we construct dependent Pareto (γ_s, β_i) severities with the joint survivor function given as:

$$S_{X_1,\ldots,X_i}(t_1,\ldots,t_i) = (1 + \frac{1}{\beta_1}t_1 + \ldots + \frac{1}{\beta_i}t_i)^{-\gamma_s}, \tag{1.3}$$

and dependent Negative Binomial (γ_f, λ_i) frequencies with the joint probability generating function described as

$$P_{N_1,\ldots,N_i}(t_1,\ldots,t_i) = [1 - \lambda_1(t_1 - 1) - \ldots - \lambda_k(t_i - 1)]^{-\gamma_f}. \tag{1.4}$$

A formal description of the above-mentioned mixture models is presented in Appendix A.

1.5 Implementation and Numerical Results

Table 1 shows summary statistics about assumed Pareto severity and Negative Binomial frequency of operational losses for each class of risk. Both severity distributions are heavy-tailed, skewed to the right and have high standard deviations. Severity distribution of the second class is more heavy-tailed than severity distribution of the first class of risk. First class of risk has higher loss occurrence than a second class. Similar to severities, frequency distributions are skewed to the right. Figure 2 provides a graphical representation of loss distributions under consideration.

We consider the following *cases* of loss dependence (see Table 2): loss frequencies are independent and so are loss severities (Case I); loss frequencies are dependent and loss severities are independent (Case II); loss frequencies are dependent and so are loss severities (Case III and Case IV). As a graphical illustration of dependence, Figure 3 shows scatter plots of severities and frequencies in Case III. One can see that loss severities are characterized by a low level of dependence (values of severities are spread) and loss frequencies are characterized by a high level of dependence (values of frequencies are concentrated along the diagonal).

We implement a numerical evaluation of the capital charge using the compound model and Monte Carlo simulation approach. We simulate aggregate losses on the level of class of risk and of a bank 200 times and therefore obtain 200 values of CaRs. Figure 4 illustrates the idea of our computations. In the LDA, first we compute CaR values for each class of risk and then sum these values up to arrive to the CaR of a bank. In contrast, in our model, the CaR of a bank is computed using aggregate losses on the bank's level directly.

In the regulatory setting, values of capital charge are defined as VaR. We demonstrate results of our computations if instead, the ES risk measure is used. Table 3 shows some summary statistics of distribution of simulated CaRs. The size of the capital charge is about 3 times higher when CaR is computed as ES versus VaR. Independently of the risk measure used for capital allocations, in all four cases of loss dependence, the values of capital charge obtained based on the LDA approach are higher than values obtained via the model we suggest. This finding totally supports the hypothesis about overestimation of the CaR computed via the regulatory LDA unless the underlying loss dependence is perfect. Another important result, which is in line with this hypothesis, is that the closer loss dependence between risk classes approaches one, the smaller the difference between CaRs computed in these two models (see Figure 5). Implementation of the model results in the following reduction in the values of CaR computed as VaR when they are compared with those obtained in the LDA: 18.51% in the first case, 15.39% in the second case, 7.86% in the third case and 2.78% in the fourth case. The highest level of capital reduction, 18.51%, as we expected, is in the independence case. Similar level of capital reduction is observed in the values of CaR when they are computed using the ES risk measure: from 18.05% in the first case to 3.39% in the fourth case (see Table 5 Panel A).

The above-mentioned results are obtained for simulated values of CaR when the true parameters of loss distributions are used. In this study, we account for the fact that in practice, the true parameters of the loss severity and loss frequency distributions are unknown and have to be estimated. To address this issue, we implement MLE estimation of parameters of the distribution of simulated loss frequencies and simulated loss severities correspondingly. These estimates should asymptotically follow Gaussian distribution. Using this property, we simulate 200 values of the severity and frequency parameters according to their approximate distribution and compute values of the capital charge using aggregate losses generated for each path of parameter estimates. This provides a probability distribution of the CaR based on the estimated values of the parameters.

Table 4 displays the results of our computations when the *estimates* of the parameters are used. Similar to the case with true parameters, the values of capital charge obtained based on the LDA approach are higher than values obtained via the model we suggest and the highest level of capital reduction is when the aggregate losses are independent. This result holds for the CaR computed as both VaR and ES (see Table 5 Panel B).

Finally, Table 5 demonstrates that introduction of dependent severities has a stronger effect on the reduction of capital at risk if compared to introduction of dependent frequencies.

To evaluate the accuracy of our computations, we measure possible underestimation of the values of the CaR when estimated versus true parameters of the loss frequency and severity distributions are used. We define the probability p of underestimation in the following way:

$$\Pr\{\widehat{\mathrm{CaR}}(\alpha) \leq (1-p) \times \mathrm{CaR}(\alpha)\} = 1 - CL, \tag{1.5}$$

where $\widehat{\mathrm{CaR}}(\alpha)$ and $\mathrm{CaR}(\alpha)$ are the capital charges obtained using the estimated and true parameters correspondingly and CL stands for the confidence level. The expression in (1.5) can be interpreted in the following way: only in $(1 - CL)$ percent of the cases, the value of the CaR is underestimated by more than p percent. The values of the CaR computed using the estimates of the parameters are exposed to average underestimation of 13.23% for 90% confidence level. The underestimation is stronger when the CaR is computed as ES. We believe that in addition to non-triviality of adequate estimation of sparse data in the tail of the distribution, the accuracy of ES for such a high quantile as 99.9% is negatively affected by a small sample size of the data in the tail.

Figure 7 provides a graphical illustration of the accuracy of the estimation of the CaR computed as VaR as well as ES. It shows empirical cumulative distribution functions of the values of CaR obtained in the model for both the true and estimated parameters.

1.6 Conclusions

This paper addresses the issue of dependence between operational losses and how it can be accounted for in the value of capital at risk for operational losses of a bank. In contrast to the Loss Distribution Approach described by regulators in Basel II, our model accounts for underlying dependence between aggregate losses of different classes of risk.

Advancing previous research, the measurement of the above-mentioned dependence accounts for *both* dependence between loss frequencies and dependence between loss severities. Though the latter type of dependence has been suggested in the literature, it has never been computationally incorporated into existing models, leaving this issue for future research. By accounting for these two sources of dependence, the approach under consideration helps to decrease inaccuracy in the measurement of bank's exposure to operational risk.

We implement our computations of a capital charge when it is defined as VaR (regulatory setting) as well as ES measure. The latter measure is not suggested by regulators. However, given its coherence property and that it reflects a possible exposure to extreme losses better than VaR, expected shortfall has advantageous implications on the capital charge for operational risk.

The model implementation shows that, for different levels of loss dependence, values of the CaR obtained based on the LDA approach are higher than values obtained via our model. This result reveals that under certain conditions, the LDA overestimates the values of CaR and that correct accounting for loss dependence has an effect on the amount of capital charge. This finding holds for the CaR computed both as VaR and as ES.

The results of this paper show that accounting for loss dependence in the value of CaR improves its accuracy. This is important for banks and regulators because for banks, the capital charge should reflect their true operational risk exposure. For regulators, the capital charge that is correctly computed across all banks would help to preserve stability in the banking and financial sectors of the economy.

Appendix A: Dependence Modelling

Dependent severities: the common Exponential-Gamma mixture model

Consider k continuous random variables $X_1, \ldots, X_k$, each representing individual operational losses of different classes of risk. Assuming that there exists a random parameter Θ such that $(X_i|\Theta = \theta_s)$ follows an exponential distribution with the parameter θ_s, the conditional survivor function of X_i is

$$S_{X_i|\Theta}(t_i|\theta_s) = Pr(X_i > t_i|\Theta = \theta_s) = e^{-\theta_s t_i}, \quad i = 1, \ldots, k.$$

We further assume that Θ has a Gamma probability density function given by

$$\pi(\theta_s) = \frac{\theta_s^{\gamma_s - 1} exp(-\theta_s \beta)}{\Gamma(\gamma_s)\beta^{-\gamma_s}} \ \theta_s > 0,$$

or, in other words, Θ ~Gamma$(\gamma_s, \frac{1}{\beta})$.

The moment generating function M_Θ is given by

$$M_\Theta(z) = (1 - \tfrac{1}{\beta} z)^{-\gamma_s}.$$

For any given $\Theta = \theta_s$, the variables $(X_i|\Theta = \theta_s)$, $i = 1, \ldots, k$, are conditionally independent and have a conditional joint survivor function

$$S_{X_1,\ldots,X_k|\Theta}(t_1, \ldots, t_k|\theta_s) = Pr(X_1 > t_1, \ldots, X_k > t_k|\Theta = \theta_s) = e^{-\theta_s(t_1 + \ldots + t_k)}.$$

At the same time, unconditionally, X_i, $i = 1, \ldots, k$ are jointly dependent as they depend upon the same random parameter θ_s. The unconditional joint survivor function for $X_1, \ldots, X_k$ is

$$\begin{aligned} S_{X_1,\ldots,X_k}(t_1, \ldots, t_k) =& Pr(X_1 > t_1, \ldots, X_k > t_k) \\ =& \int_0^\infty e^{-\theta_s(t_1 + \cdots + t_k)} \pi(\theta_s) d\theta_s \\ =& M_\Theta(-t_1 - \ldots - t_k). \end{aligned}$$

Substituting the moment generating function of the Gamma distribution into the expression above, we get the following survivor function of a multivariate Pareto distribution:

$$S_{X_1,\ldots,X_k}(t_1, \ldots, t_k) = (1 + \tfrac{1}{\beta_1} t_1 + \ldots + \tfrac{1}{\beta_k} t_k)^{-\gamma_s}, \ i = 1, \ldots, k.$$

One can see from the joint survivor function that the marginal survivor distributions of the presented multivariate model are Pareto (γ_s, β_i)

$$S_{X_i}(t_i) = S_{X_1,\ldots,X_k}(t_1, \ldots, t_k)|_{t_j = 0, j \neq i} = (1 + \tfrac{1}{\beta_i} t_i)^{-\gamma_s},$$

which implies the following cumulative distribution function for X_i

$$F_{X_i}(t_i) = 1 - (\tfrac{\beta_i}{t_i + \beta_i})^{\gamma_s},$$

that is of Pareto (γ_s, β_i) distribution.

Dependent frequencies: the common Poisson-Gamma mixture model

Let us consider k discrete random variables $N_1, \ldots, N_k$. Assume that there exist a random parameter Θ such that

$$(N_i|\Theta = \theta_f) \sim \text{Poisson } (\theta_f \lambda_i),\ i = 1, \ldots, k,$$

where Θ ~Gamma$(\gamma_f, 1)$, has a probability density function $\pi(\theta_f)$ and a moment generating function $M_\Theta(z) = (1-z)^{-\gamma_f}$.

For any given $\Theta = \theta_f$, the variables $(N_i|\Theta = \theta_f)$ are independent and Poisson $(\theta_f \lambda_i)$ distributed with a conditional joint probability generating function (pgf) given as

$$P_{N_1,\ldots,N_k|\Theta}(t_1, \ldots, t_k|\theta_f) = E[t_1^{N_1}, \ldots, t_k^{N_k}|\Theta = \theta_f] = e^{\theta_f[\lambda_1(t_1-1)+\cdots+\lambda_k(t_k-1)]}.$$

At the same time, unconditionally, $N_1, \ldots, N_k$ are jointly dependent and have the following joint unconditional probability generating function:

$$\begin{aligned}
P_{N_1,\ldots,N_k}(t_1, \ldots, t_k) =& E_\Theta[E(t_1^{N_1}, \ldots, t_k^{N_k}|\Theta)] = \int_0^\infty e^{\theta_f[\lambda_1(t_1-1)+\ldots+\lambda_k(t_k-1)]}\pi(\theta_f)d\theta_f \\
=& M_\Theta(\lambda_1(t_1-1) + \ldots + \lambda_k(t_k-1)) \\
=& [1 - \lambda_1(t_1-1) - \ldots - \lambda_k(t_k-1)]^{-\gamma_f}.
\end{aligned}$$

This joint probability generating function defines a multivariate negative binomial distribution with negative binomial margins $N_i \sim \text{NB}(\gamma_f, \lambda_i)$:

$$\begin{aligned}
P_{N_i}(t_i) =& P_{N_1,\ldots,N_k}(t_1, \ldots, t_k)|_{t_j=1, i\neq j} \\
=& M_\Theta(\lambda_1(t_1-1) + \cdots + \lambda_k(t_k-1))|_{t_j=1, i\neq j} \\
=& M_\Theta(\lambda_i(t_i-1)) = (1 - \lambda_i(t_i-1))^{-\gamma_f}.
\end{aligned}$$

Appendix B: Figures

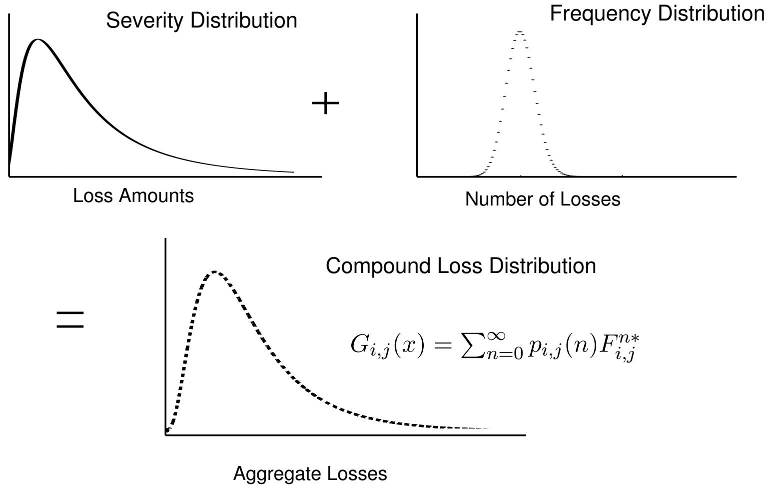


Figure 1. The Loss Distribution Approach. The figure displays the idea of the LDA. First severity and frequency distributions are modelled. Second, the loss aggregate distribution is computed via the compound model and using aforementioned distributions as inputs.

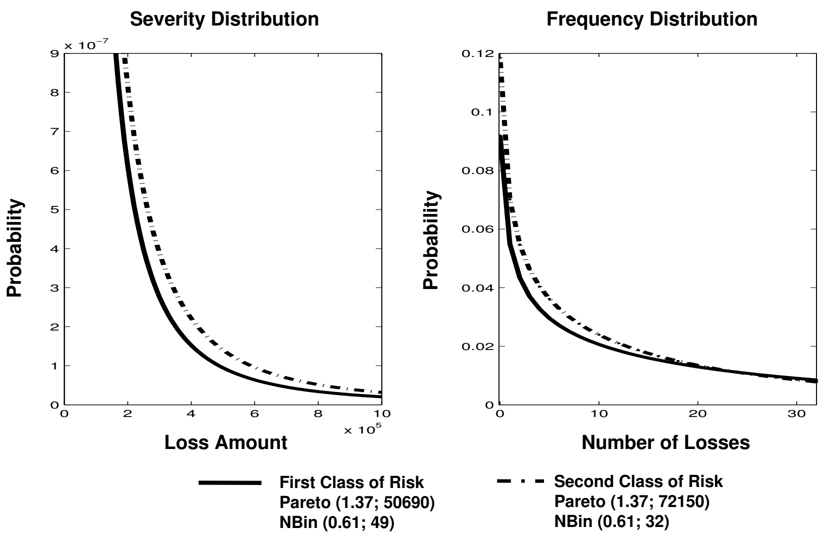


Figure 2. Loss Distributions. The figure displays severity and frequency distributions of each class of risk.

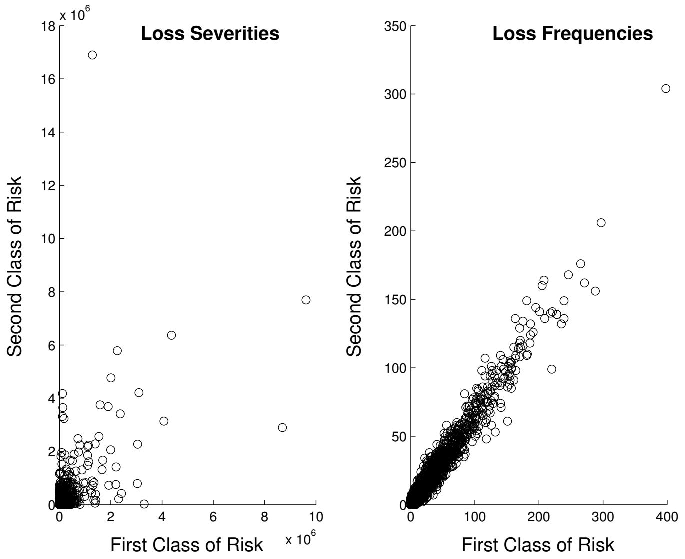


Figure 3. Loss Dependence. The figure shows two samples: a sample of dependent loss severities and a sample of dependent loss frequencies.

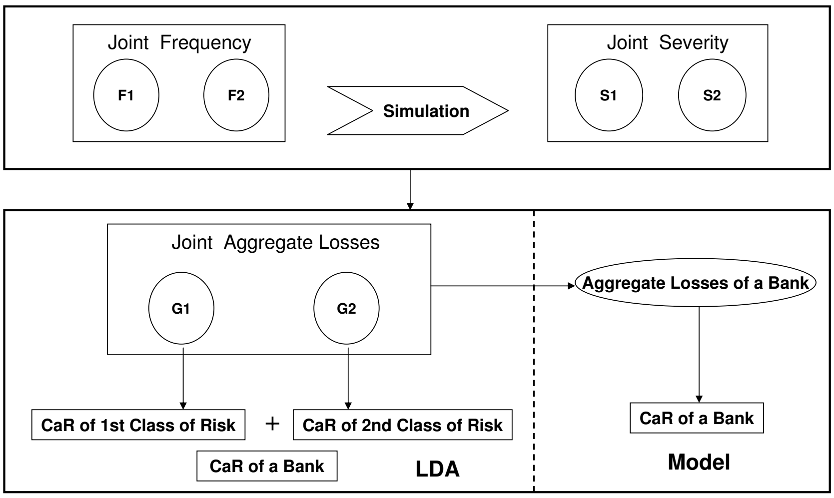


Figure 4. LDA vs.the Model. The figure illustrates the idea of CaR computations implemented based on the LDA and using the model we suggest.

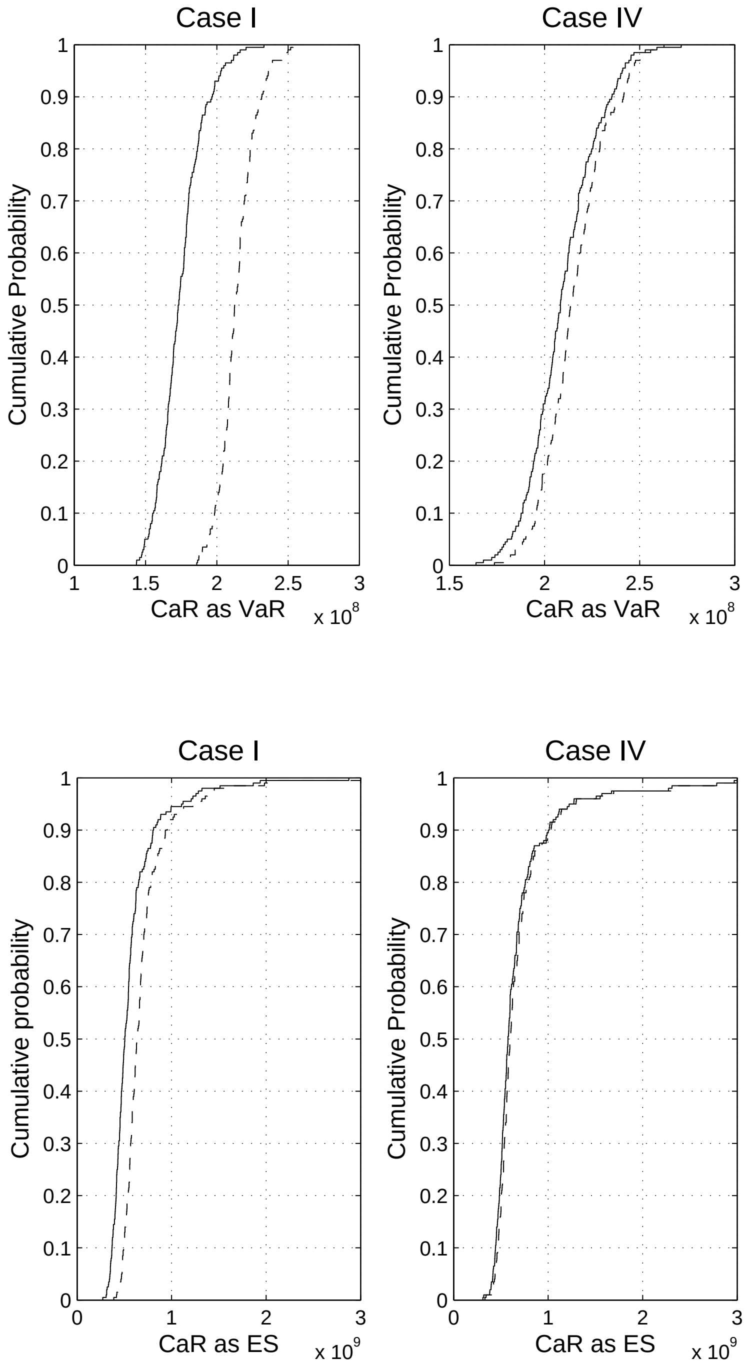


Figure 5. Capital at Risk: LDA vs. the Model: True Parameters. The figure shows empirical cumulative distribution functions of the values of the CaR obtained using the LDA (dashed line) and using the model we suggest (solid line). The results are given for two cases of loss dependence: Case I (independence) and Case IV (high dependence). The CaR is computed as $VaR_{99.9\%}$ (upper panel) and as $ES_{99.9\%}$ (lower panel).

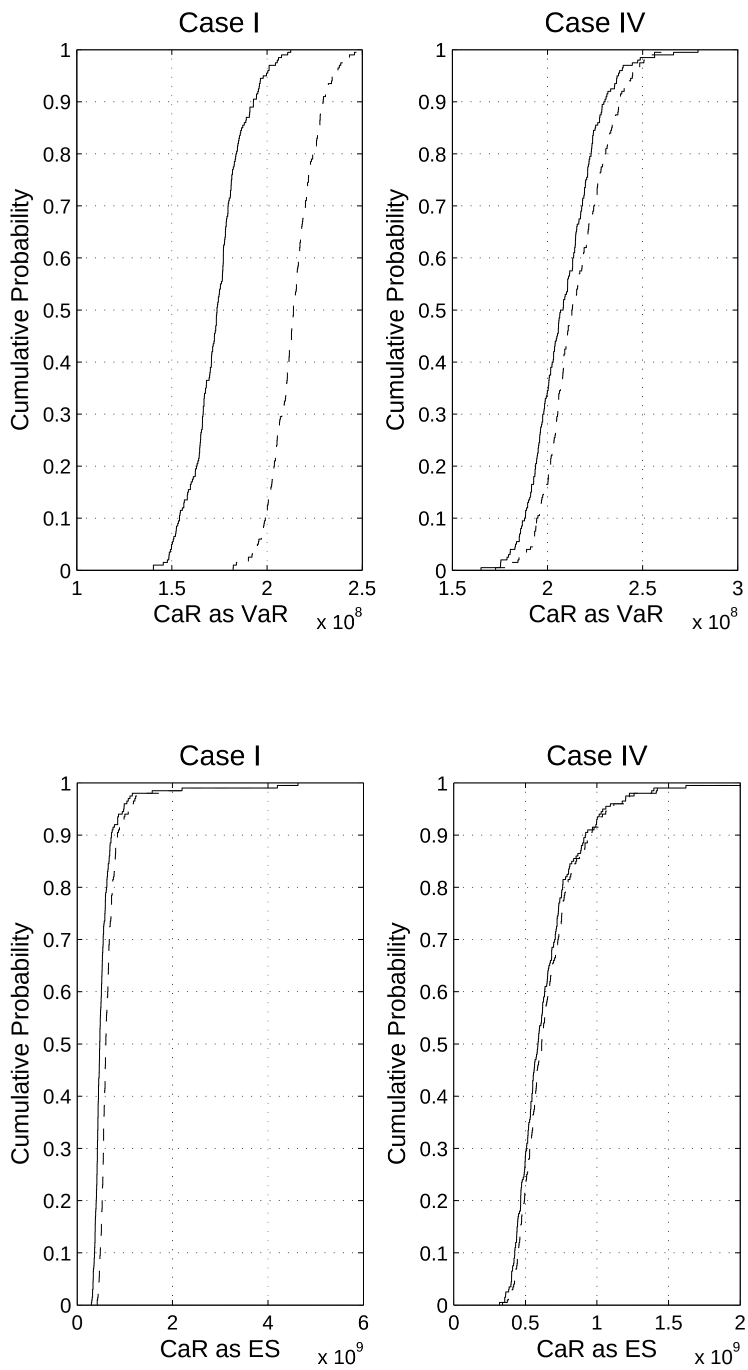


Figure 6. Capital at Risk: LDA vs. the Model: Estimated Parameters. The figure shows empirical cumulative distribution functions of the values of the CaR obtained using the LDA (dashed line) and using the model we suggest (solid line). The results are given for two cases of loss dependence: Case I (independence) and Case IV (high dependence). The CaR is computed as $VaR_{99.9\%}$ (upper panel) and as $ES_{99.9\%}$ (lower panel).

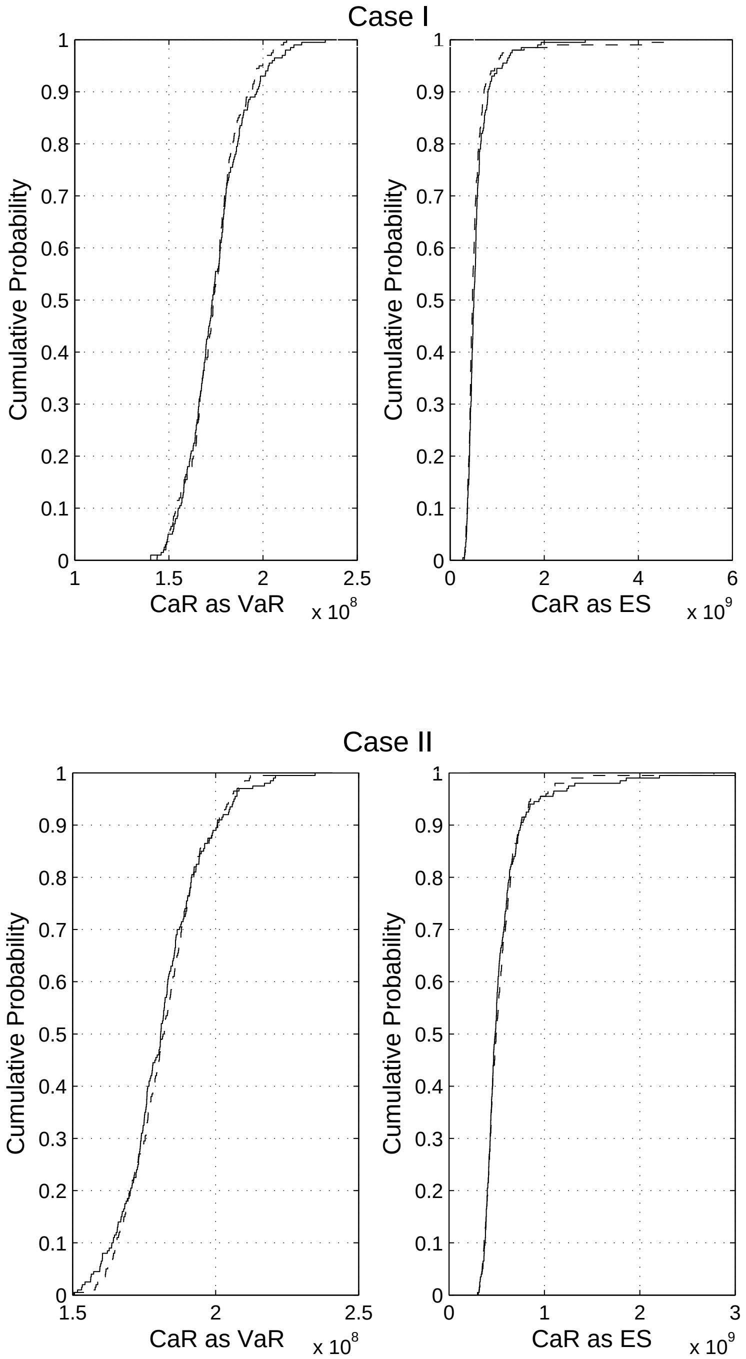
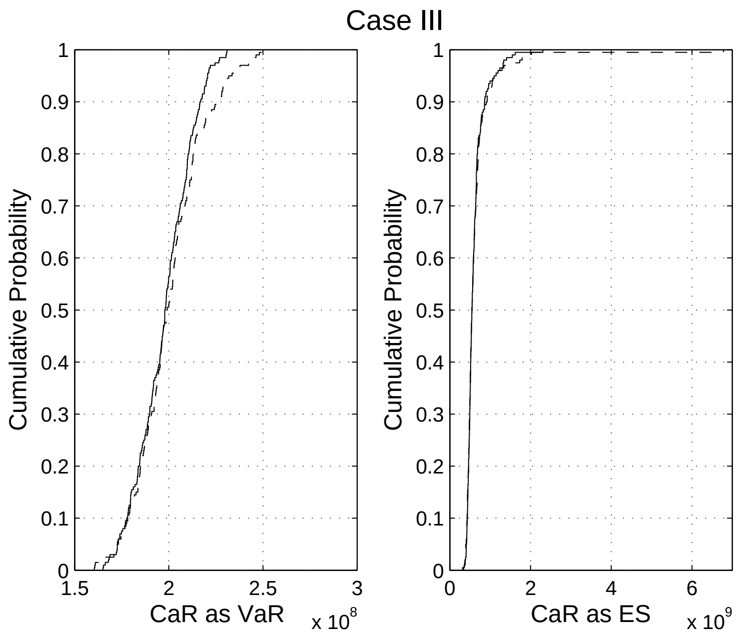


Figure 7. Capital at Risk: Accuracy of Estimation. The figure shows empirical cumulative distribution functions of the values of the CaR obtained using the model we suggest. The results are given for each case of loss dependence and using the true (solid line) and estimated (dashed line) parameters of the loss severity and loss frequency distributions. The CaR is computed as $VaR_{99.9\%}$ (left panel) and as $ES_{99.9\%}$ (right panel).



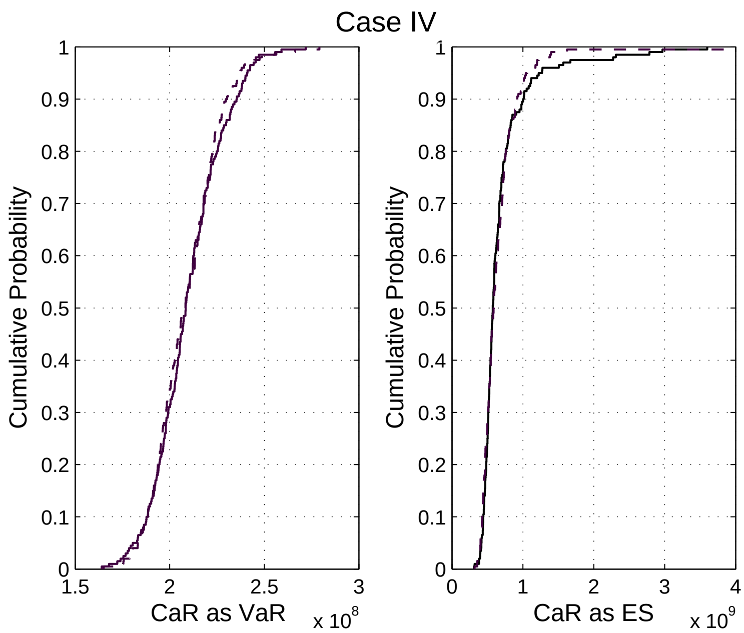


Figure 7 (Cont'd.). Capital at Risk: Accuracy of Estimation. The figure shows empirical cumulative distribution functions of the values of the CaR obtained using the model we suggest. The results are given for each case of loss dependence and using the true (solid line) and estimated (dashed line) parameters of the loss severity and loss frequency distributions. The CaR is computed as $VaR_{99.9\%}$ (left panel) and as $ES_{99.9\%}$ (right panel).

Appendix C: Tables

Panel A: Operational loss severity

Class of risk	Mean	Standard Deviation	Skewness	Kurtosis
I: Pareto $(1.37, 50690)$	140	1300	22	649
II: Pareto $(1.37, 72150)$	194	1584	21	647

Panel B: Operational loss frequency

Class of risk	Mean	Standard Deviation	Skewness	Kurtosis
I: NBin $(0.61, 49)$	31	41	2.4	10.4
II: NBin $(0.61, 32)$	20	27	2.5	10.6

Table 1. Moments of Loss Distributions. The table provides a summary statistics about operational loss severity and frequency distributions. Mean and standard deviation of loss severities are given in Euro, 000. Described loss frequencies are per year.

Loss dependence	Case I	Case II	Case III	Case IV
Severity level	0.00	0.00	0.27	1.00
Frequency level	0.00	0.82	0.82	0.82
Class of risk level	0.00	0.64	0.71	0.78

Table 2. Loss Dependence. The table displays values of loss dependence measured by the Kendall's tau. In each case considered, the resulting dependence on the level of class of risk is driven by the dependence between loss frequencies and dependence between loss severities.

	LDA						Model					
	Mean	Std	Min	Max	$CI^{-}_{95\%}$	$CI^{+}_{95\%}$	Mean	Std	Min	Max	$CI^{-}_{95\%}$	$CI^{+}_{95\%}$
Panel A: Capital at Risk as $VaR_{99.9\%}$												
Case I	214,282	12,813	185,949	253,696	189,430	245,705	174,617	15,727	143,705	233,127	147,527	211,784
Case II	214,474	12,732	182,794	255,148	191,107	241,407	181,463	14,395	150,421	234,775	154,328	212,997
Case III	214,493	12,843	175,209	247,351	188,774	237,503	197,637	14,668	165,024	230,902	168,355	224,564
Case IV	215,776	16,772	173,674	262,849	184,579	250,259	209,779	18,257	164,027	271,920	175,521	245,446
Panel B: Capital at Risk as $ES_{99.9\%}$												
Case I	705,181	292,860	387,764	3,026,203	438,810	1,427,508	577,900	291,101	271,956	2,874,505	318,576	1,282,810
Case II	699,718	507,858	396,584	6,973,162	439,908	1,358,960	580,076	507,622	299,290	6,852,110	323,044	1,252,745
Case III	677,356	244,677	367,527	2,358,604	446,804	1,389,500	618,827	244,920	314,541	2,306,115	391,661	1,321,570
Case IV	711,685	408,320	330,426	3,626,527	420,864	1,692,981	687,576	407,779	308,731	3,598,881	396,218	1,671,157

Table 3. Capital at Risk: True Parameters. The table displays the values of the CaR computed based on the LDA and using our model. All computations are implemented using the true parameters of the loss severity and loss frequency distributions. For each model, some statistics and 95% confidence intervals for CaR are presented. All values are given in Euro, 000.

	LDA						Model					
	Mean	Std	Min	Max	$CI^{-}_{95\%}$	$CI^{+}_{95\%}$	Mean	Std	Min	Max	$CI^{-}_{95\%}$	$CI^{+}_{95\%}$
Panel A: Capital at Risk as $VaR_{99.9\%}$												
Case I	214,252	12,204	182,110	247,201	190,245	238,811	173,739	14,101	140,288	212,505	148,429	204,511
Case II	214,724	12,644	187,612	258,642	190,186	238,210	182,115	12,927	150,678	223,455	158,777	207,934
Case III	216,131	15,638	177,812	265,003	187,197	249,692	200,369	18,038	160,481	252,758	162,902	242,408
Case IV	215,380	16,686	172,778	264,190	185,084	248,517	208,857	17,256	165,106	279,132	178,790	244,666
Panel B: Capital at Risk as $ES_{99.9\%}$												
Case I	692,833	441,178	421,015	4,751,995	439,698	1,226,970	565,968	441,344	301,625	4,629,849	325,913	1,107,821
Case II	671,080	240,744	414,270	2,881,223	439,509	1,212,441	551,068	239,515	307,299	2,776,887	329,373	1,085,875
Case III	713,081	507,722	405,413	6,838,519	436,469	1,662,740	654,078	507,930	340,820	6,783,712	382,047	1,609,482
Case IV	694,288	515,274	340,178	7,304,207	391,548	1,227,146	669,535	514,820	318,870	7,277,954	364,614	1,201,712

Table 4. Capital at Risk: Estimated Parameters. The table displays the values of the CaR computed based on the LDA and using our model. All computations are implemented using the estimated parameters of the loss severity and loss frequency distributions. For each model, some statistics and 95% confidence intervals for CaR are presented. All values are given in Euro, 000.

	CaR as $VaR_{99.9\%}$			**CaR as** $ES_{99.9\%}$		
	LDA	Model	Reduction	LDA	Model	Reduction
	Panel A: True Parameters					
Case I	214,282	174,617	18.51 %	705,181	577,890	18.05 %
Case II	214,474	181,463	15.39 %	699,718	580,076	17.10 %
Case III	214,493	197,637	7.86 %	677,356	618,827	8.64 %
Case IV	215,776	209,779	2.78 %	711,685	687,576	3.39 %
	Panel B: Estimated Parameters					
Case I	214,252	173,739	18.91 %	692,833	565,968	18.31 %
Case II	214,724	182,116	15.19 %	671,080	551,068	17.88 %
Case III	216,131	200,369	7.29 %	713,081	654,078	8.27 %
Case IV	215,380	208,856	3.03 %	694,288	669,535	3.56 %

Table 5. Reduction in the CaR. The table describes the reduction in the values of capital charge computed in the model and when it is compared to the LDA.

Paper 2

The Impact of Terrorism on Financial Markets: An Empirical Study

2.1 Introduction

A lot of research about terrorism has been done in the fields of sociology, political science and history. As to economics and finance, terrorism has not received much attention from researchers until recently. The effect of the impact of the 9/11 terrorist attacks on stock markets as well as that of more recent attacks in Madrid in 2004 and in London in 2005 has revealed that terrorism risk is a new type of catastrophic risk that investors and financial institutions may be facing. The intention of this paper is to provide a deeper understanding of the impact of this risk on the behavior of different financial markets. When studying the impact, we look at global, regional, national and industrial market levels. In addition, we compare the impact of terrorist events on financial markets with the impact of other extreme events such as financial crashes and natural catastrophes.

Among existing research, this empirical paper is one of the very few (see Chen and Siembs (2004), Eldor and Melnick (2004), Karolyi and Martell (2006)) that study the link between terrorism and the behavior of stock markets. It is also the first one that analyzes the impact on the bond and commodity markets. In contrast to impact studies that often employ only event-study methodology, in this work we investigate the impact of terrorism using other methods as well. These are non-parametric methodology and a filtered GARCH with the Extreme Value Theory approach. Both methods are standard econometric tools. However, the way we apply them in this work is original.

The findings of our empirical investigation are useful for investors, insurance and re-insurance businesses, banks and government agencies. The empirical results of this work can give insights into portfolio diversification strategy with respect to the risk of terrorism. To diversify this risk, investors may invest in industries and markets that are less 'sensitive' in a negative way or, alternatively, in those that exhibit a positive reaction to terrorist events. Regarding the latter aspect, we check the impact of terrorist attacks on the bond and commodity/gold markets that are usually considered as those providing 'safe-haven' investment opportunities.

To study the impact of terrorism on financial markets empirically, we look at the effect of 77 terrorist attacks that occurred in 25 countries over an 11 year time period (see Table 1). We look at global, European, American, and Swiss stock markets as well as the insurance, banking, travel, airline, defense, pharma/biotech, aero/defense and oil/gas industrial stock indices. In relation to other markets, we look at the US, European and World bond indices as well as at the global commodity and gold markets. When comparing the impact of terrorist events on these financial markets with the impact of other extreme events, we analyze the impact of 4 financial crashes and 19 natural catastrophes that occurred during the 11 year period under our consideration.

When analyzing terrorism risk, data about terrorist attacks are of significant value. To implement our analysis, we construct a database of terrorist events that occurred around the world. We use publicly available information about terrorist attacks. Since limited availability of historical information about terrorist events is often considered as a restriction to modelling terrorism risk, we consider the data collection as one of the first important steps in approaching the topic. In addition, and more importantly, we aim at incorporating results about the impact of terrorist attacks on different financial markets indices into our database (see Table 6). Adding a financial component to the database makes it original and more valuable.

The results of our work are as follows. Around two thirds of the terrorist attacks considered lead to significant a negative impact on at least one stock market under consideration. The Swiss stock market is affected by the highest number of attacks while the American stock market by the lowest number. The insurance sector and the airline industry exhibit the highest susceptibility to terrorism, while the banking industry is the least sensitive[1]. This is in contrast to financial crashes that demonstrate a strong negative impact on the banking sector. The analysis of the impact on the aero/defense, pharma/biotech and oil/gas sectors shows both positive and negative reactions. These stock markets behave similarly in case of natural disasters and financial crashes.

As with terrorist events, natural catastrophes cause both positive and negative return movements in the commodity/gold and bond markets. The gold index is affected by a lower number of events compared to the commodity index, implying less sensitivity of a former market to natural disasters. Finally, among bond markets considered, the U.S. government bond market shows the lowest impact from terrorist attacks, natural catastrophes and financial crashes.

As to the strength of the impact, terrorist attacks and financial crashes cause event-day return movements that are mostly extreme, with the strength of the impact declining in the post-event period. This implies that though markets perceive these events as unusual, they do not see their effects as long-lasting. As to natural catastrophes, the negative impact is more often observed in the post-event period. This can be attributed to the fact that markets need more time to evaluate the long-term impact of such events. In addition, natural disasters can last for several days and therefore the impact is more likely to be evaluated during the period following the event.

The results of this paper suggest several diversification strategies for dealing with terrorism risk. If concerned about this risk, investors should consider holding two groups of assets: those that are likely to react positively to terrorist attacks or those that have little or no negative sensitivity to this risk. In the first case, a U.S. Government bond index is the safest choice followed by such industry stocks as aero/defense and pharma/biotech. However, given that these stock markets may also exhibit a negative response, investing

[1]Note that the banking sector was affected negatively in the case of the 9/11 attacks. However, this event was exceptional in terms of its magnitude and place of occurrence (Manhattan, the financial center).

in these industries as a diversification strategy against terrorist attacks may not always work. In the second case, banking stock index may be good for investment. Note that, though this stock index is least sensitive to terrorist attacks, it exhibits significant negative return movements associated with financial crashes.

In relation to terrorist attacks, investing in a composite commodity index is preferable to investing in gold only. This is because, with terrorist events, the gold market reacts negatively more often than positively. In addition, when it is compared to the commodity market in general, the negative impact on the gold market is more long-lasting. At the same time, the commodity market also shows a short-term negative reaction to some terrorist events. This implies that investing in gold and commodity markets may not always provide a good hedge.

Another possible way to reduce negative exposure to terrorist events would be to avoid investing in insurance, travel and airline industry stocks. Note that insurance and airline industries show high negative sensitivity not only to terrorist attacks but also to financial crashes and natural disasters. This implies that by keeping these stocks in their portfolio, investors may end up increasing their risk of loss if further terrorist attacks arise.

Finally, the response of financial markets to terrorist events suggests several strategies of trading derivatives. For example, investors can hold long positions in put options on the industry stocks that may react negatively to terrorist events (for example, airline and insurance industry stocks). Or, alternatively, they can invest in call options, where the underlying asset is a U.S. Government bond index.

2.2 Related Research

Analysis of the existing literature on the impact of terrorism on financial markets shows that most of the research has a descriptive character and focuses on the impact of very few terrorist events (often only those which occurred on September 11, 2001). Johnston and Nedelescu (2005) examine cases where financial markets are directly or indirectly affected by terrorist acts. They review the reaction of the markets to the 9/11 attacks in the U.S. and attacks in Madrid in March, 2004. The main conclusion of their study is that financial markets are not only confronted with major disruptions caused by the massive damage to property and communication systems, but also with high levels of uncertainty and market volatility, especially in the case of the 9/11 attacks in New York (IMF (2001)). However, there are some differences in the stock market reaction to these two terrorist events. While attacks in Madrid were perceived as mostly having a regional effect, those in New York were seen as having repercussions on the global financial system[2]. The authors view the timing of attacks as a possible explanation for different impacts. Attacks in New York occurred in a period of economic downtown. In contrast, the attacks in Spain happened when the world economy was experiencing growth. We think that the difference in the impact can also be explained by looking at the targets of the attacks. The 9/11 attacks happened in Manhattan, the financial center, while the bombings in Madrid were targeted at a transport system.

Further evidence of the impact of terrorism on financial markets is offered by some impact studies. Among existing literature, this paper is most closely related to that by Chen and Wei (2005). These authors examine the U.S. capital market reaction to 7 terrorist and 7 military attacks over the period 1915-2001, using an event study approach. They apply their analysis to some other capital markets as well, but focus on the impact of only two events: the 9/11 terrorist attacks and Iraq's invasion of Kuwait in 1990. They find that U.S. capital markets rebound and stabilize quicker after these two events compared to other markets, and that US markets are more resilient now than in the past, which they explain by the strength of the banking and financial sectors in the U.S. One of the main conclusions of their paper is that financial markets are efficient in absorbing the shocks caused by terrorist attacks and can continue to function in an effective way. Compared with the work by Chen and Siembs (2004), this study covers a much wider range of terrorist events (77 versus 7) and applies not only the traditional event-study approach but more rigorous econometric techniques such as the non-parametric methodology and the filtered GARCH-Extreme Value Theory approach.

[2]The major worldwide equity markets experienced sharp and rapid declines, demonstrating that market participants perceived the 9/11 event as a global shock. In contrast, the 2004 terrorist bombings in Madrid had much less effect on the financial markets. The Dow Jones EURO STOXX fell by about 3% on March 11, and continued to drop during the following days but had recovered almost completely by the end of the month. Similarly, after a small decline, the Standard and Poors 500 returned to pre-March 11 levels in less than a month (Johnston and Nedelescu (2005)).

Eldor and Melnick (2004) study how stock and foreign exchange markets react to terrorism in Israel. The authors consider 639 terror attacks during the period from 1990 to 2003 and distinguish the data by location, target, type of attack and number of casualties. They show empirically that terrorism has a permanent negative effect on the stock market but not on the foreign currency market. They conclude that these markets are efficient in incorporating news about terrorist attacks and that there is no evidence that markets have become desensitized to the terror over time.

Several studies consider the effect of the September 11^{th} attacks alone on the stock market. The paper by Carter and Simkins (2001) examines the impact of this event on airline stock returns. They test whether market reaction on the first trading day after the attack is the same for each airline or, alternatively, whether it distinguishes among airlines based on firm characteristics. They find that market differentiates among various airlines based on their ability to cover short-term obligations as measured by a ratio of cash and equivalents to total assets. According to their study, airlines with low liquidity are penalized the most. No statistical significance is found for such firm characteristics as size, leverage and firm performance.

Other research focuses on the economic consequences and associated costs of terrorism. In their papers, Abadie and Gardeazabal (2003), Abadie and Gardeazabal (2005) study the effects of terrorism on economic activity. Krugman (2003) refers to direct economic damage costs, the budget costs of government responses to terrorism and the cost imposed by the way people respond to fears of terrorism. The long-term economic impact of terrorism is also studied by Karolyi and Martell (2006). Authors examine the stock price impact of terrorist attacks in which traded firms are targets. They find that the impact of attacks differs according to the home country of the target firm and the country in which the incident occurs. They conclude that in countries that are wealthier and more democratic, attacks are associated with larger share price reactions. According to Raby (2003), airline, travel, tourism, accommodation, restaurant, postal and insurance industries are particularly susceptible to increased terrorism risks. Regions and economies where these industries are concentrated are likely to suffer most from falls in output and employment. Supporting this point, there are several papers that study the relationship between terrorism and tourism (see Enders and Sandler (1991), Darkos and Kutan (2001), Enders, Sandler, and Parise (1992), Pizam and G.Smith (2000)). They show that for countries like Spain, Greece, Austria, Turkey and Israel, terrorism has a substantial effect on tourism.

Some papers discuss ways of modelling and measuring terrorism risk. Frey and Luechinger (2003), for example, discuss measurement issues and review different methods that can be used in this context. The paper by Pate-Cornell and S.Guikema (2002) describes the model for setting priorities among threats and among countermeasures to terrorism based on probabilistic risk analysis, decision analysis, and elements of game theory. Their model accounts for the probabilities of different scenarios, the objectives of both the terrorists and the U.S, and the dynamic competition between them.

There is a strand of literature that discusses financial instruments that can be used to hedge terrorism risk: terrorism catastrophe bonds (CAT bonds) and 'terrorism futures'. In their paper, Bantwal and Kunreuther (2000) discuss CAT bonds in the context of natural hazard events. However, given similarities between these events and terrorist attacks, the paper's conclusions are useful in the context of terrorism risk. The authors identify factors that explain a relative reluctance of institutional investors to enter the market of CAT bonds. Applied to terrorism risk, such bonds can be quite expensive since capital market investors require high premiums to compensate for the uncertainty about this risk. Among other factors are investors' unfamiliarity with these new instruments and difficulty with pricing. At the same time, terrorism CAT bonds still have the potential to be attractive alternative risk transfer instruments when exposure to this risk is mixed with exposure to other catastrophe risks[3].

Since financial markets are quite powerful in terms of information aggregation, they may be good predictors of terrorist events. The idea of 'terrorism futures' market has been

[3]There were two transactions that involved securitization of terror-related losses: the FIFA transaction, Golden Goal Finance Ltd., initiated by FIFA governing body in September 2003 (OECD (2005)) and the Vita Capital transaction implemented by Swiss Re in December 2003 (Brauner and Galey (2003)). In the first case, the CAT bond was aimed at covering revenue losses that would arise in the event of cancelation of the 18th FIFA World Cup scheduled to be held in the summer of 2006 in Germany. This transaction transferred the risk of the sporting event being canceled due to natural/ man-made catastrophes and terrorist events. In the second case, the extreme mortality risk was hedged by means of the catastrophe-indexed notes. These instruments were linked to a rise in the annual mortality index from natural disasters, epidemics, war and terrorist attacks. While terrorism risk in these transactions was not a main risk under coverage, the over-subscription for these instruments demonstrated that investors were prepared to buy bonds with default tied to some level of terrorism risk.

actively discussed in the press since 2003[4]. The authors of the paper 'Prediction Markets', Wolfers and Zitzewitz (2004) comment on the potential of this market in the following way: 'Betting on human lives seems ethically questionable. Yet if it helps save lives, surely the moral questions are mitigated'. The main concern about 'terrorism futures' relates to possible speculative trading through Internet accounts and incentives for terrorists to commit acts of terror. Despite the above-mentioned concern, we believe, purely from the research point, that the development of terrorism futures as financial instruments and their pricing is a challenging and interesting task.

In this paper, we compare the impact of terrorist events with the effect of other extreme events like natural disasters and financial crashes. Johansen and Sornette (2006), Johansen, Sornette, and Ledoit (1999), Sornette (2004) offer an interesting classification of crashes (large declines in price caused by shocks) as either events of an endogenous origin associated with preceding speculative bubbles with log-periodic power law signatures (LPPS) or as events of exogenous origin associated with the market response to external shocks. They find that no striking news can be associated with outliers preceded by LPPS, while outliers without LPPS have been triggered by news surprise. In other words, exogenous crashes can be attributed to extraordinary important external perturbations in the form of news. The authors conclude that most of the crashes are endogenous and can be seen as the natural deaths of self-organized bubbles that give rise to specific precursory signatures, LPPS in particular. They define the 9/11 attacks and the coup in the Soviet Union in 1991 as exogenous shocks and such events as the "dot.com" crash in 2000 and the crash of October 1987 as endogenous shocks. The research implemented by the authors does not have a direct link with the current work. However, there are several interesting insights that one can take from their studies. First, methodologically, implemented analysis relies on statistical properties of large losses that are, in turn, studied by looking at drawdowns rather than at return movements. Second, a proposed theoretical framework for critical crashes shows the link between crashes in stock markets and critical behavior of complex systems. According to the authors, it is possible to identify

[4]In 2003, the Defense Advanced Research Projects Agency (DARPA) initiated the program called the Futures Markets Applied to Prediction (FutureMAP). The aim was to set up an online futures exchange, where people could bet on terrorist activities in the Middle East. However, the program faced a lot of criticism from government and was abandoned.

clear signs of near-critical behavior many years before the crash and to 'predict' in the endogenous case the date when the system will become critical, which in turn, is very close to a realized crash date. Finally, they test empirically two alternatives to describe crashes: one by means of the EVT, and another by the rational expectation model they develop. They conclude that large crashes are outliers that are better described by the latter model.

2.3 Terrorism Risk

Schmidt and A.Jongman (1988) present 109 different definitions of terrorism, which are obtained in a survey of leading academics in the field. In all these definitions the following words appear most often: violence and force (83.5%); political (65%); fear, emphasis on terror (51%); threats (47%); psychological effects and anticipated reactions (41.5%); discrepancy between the targets and the victims (37.5%); intentional, planned, systematic, organized action (32%); methods of combat, strategy, tactics (30.5%) Boaz (1998). One of the largest reinsurance companies, Swiss Re defines terrorism in the following way: 'Terrorism means an act or threat of violence or an act harmful to human life, tangible or intangible property or infrastructure with the intention or effect to influence any government or to put the public or any segment of the public in fear' (Brauner and Galey (2003)).

From an economic and financial standpoint, terrorism has been described as having several negative effects, such as reduction in the human and physical capital of a country, increased costs of financial and other counter-terrorism regulations, vulnerability of critical infrastructure (power plants, nuclear facilities, chemical factories, bridges, pipelines and water supply), increased financial instability, destruction in market infrastructure and operations and decrease in investor confidence (see Johnston and Nedelescu (2005), Bonturi, Koen, and Lenain (2002), Abadie and Gardeazabal (2005)). Because of enormous loss potential, terrorism risk may put high financial demands on insurance and reinsurance businesses and induce high insurance premiums[5]. As of today, insurance companies

[5] As an example, the terrorist attacks of September 11, 2001 had a wide range of negative consequences on the local and global levels. Nearly $3,050$ people were killed. Damage inflicted is currently estimated at around 80 bn USD , about half of which was insured; i.e. the most costly event in the history of insurance (Brauner and

mostly transfer this risk to reinsurance businesses. When dealing with terrorism risk, the main challenge for both types of financial institutions lies in its quantification. Even though some models have been proposed to handle this problem, existing approaches are linked with catastrophe modelling. In many ways, terrorism risk is similar to the risk of natural hazards such as floods, earthquakes, hurricanes and storms. In all these events, there is enormous loss potential and these events can affect entire economies. For example, the 9/11 attacks have evidenced that terrorism is potentially a catastrophic risk (see Table 2). At the same time, there are several crucial differences between terrorist attacks and the above-mentioned extreme events. Unlike terrorist attacks, catastrophes are natural events that occur without intent and their possible place of occurrence may possibly be predicted with less difficulty. Terrorist events are characterized by dynamic uncertainty in terms of their type (a suicide bombing, an armed assault, kidnapping etc.), their target (military, personnel, government, facilities etc.) and place and time of occurrence. Terrorists may respond to security measures by shifting their attention to new targets, changing the type of terrorist attack and the place and time of its occurrence. In other words, they behave strategically. In contrast, the actions that can be taken to reduce the damage from possible natural disasters do not affect the probability and the place of occurrence of these events.

The main challenge is to predict the likelihood and the financial consequences of terrorist attacks and quantify a possible exposure to terrorism risk. In addition, when modelling this risk, analysts are faced with a limited availability of historical data on terrorism losses. But even if these data were easily accessible, it would not necessarily reflect the changing expectations of planned terrorist activities today. In contrast, probabilities and consequences of natural hazards can be modelled and quantified more easily using well-defined methods and historical data[6]. Because of the above-mentioned characteristics, it is much

Galey 2003). Financial markets responded with sharp and rapid declines in the major worldwide equity markets, demonstrating that market participants perceived the 9/11 event as a global shock. The decline in European and global stock markets was quite strong: between September 11 and September 21, the Dow Jones EURO STOXX index was down 17.3%, FTSE 100 fell by 11.9%, MSCI Asia by 14.4% and MSCI Latin America by 15.4%. In the credit market, the LIBOR overnight was down 129 basis points and three months Treasury bill 103 basis points, IMF (2001).

[6]Catastrophe modelers develop the first generation of models to provide insurers with reliable estimates of losses from terrorist attacks. These models help to reduce uncertainty in estimates of terrorism risk (Kunreuther, E.Michel-Kerjan, and Porter (2003)).

more difficult to manage terrorism risk than the risk of natural hazards. This, in turn, calls for more studies of terrorism risk, and our work is the first that analyzes differences in the impact of terrorist attacks and natural disasters on the behavior of financial markets.

The above-mentioned uncertainties associated with terrorism risk and problems with its insurability[7] have induced government intervention into insurance markets in the OECD countries[8]. In addition to actions undertaken, other ways to manage terrorism risk, namely terrorism CAT bonds and 'terrorism futures' have been discussed by practitioners and academics.

2.4 Empirical Analysis

2.4.1 Hypotheses

When implementing our empirical study, we test the following hypotheses:

- *Hypothesis 1: Terrorist attacks do not have a significant effect on global, European, American and Swiss stock markets.*
- *Hypothesis 2: Terrorist attacks do not have a significant effect on such industry indices as insurance, travel/pleasure, airline, oil and gas, financials and banking.*
- *Hypothesis 3: Terrorist attacks do not have a significant effect on such industry indices as defense and pharmaceutical/biotechnology.*
- *Hypothesis 4: Terrorist attacks do not have a significant effect on the commodity and gold markets.*
- *Hypothesis 5: Terrorist attacks do not have a significant effect on the bond market.*
- *Hypothesis 6: Terrorist attacks do not have a significant effect on financial markets on both the event-day and in the post-event window.*

[7] Prior to the 9/11 attacks, terrorism coverage in the United States was included in most standard commercial policy packages. The private insurance market had functioned effectively because losses from terrorism had historically been small (Brauner and Galey (2003)).

[8] A good overview of U.S. and European approaches to insure catastrophe risk (which covers natural hazards and terrorism risk) with particular focus on public-private partnerships and the Terrorism Risk Insurance Act of 2002 (GAO (2005b)) can be found in a recent report issued by the United States Government Accountability Office (GAO (2005a)).

- *Hypothesis 7: Terrorist attacks do not have an effect on financial markets which is similar to that of natural catastrophes and financial crashes.*

2.4.2 Data

There are two types of data sets that we use. The first data set includes the daily prices of financial markets indices. All indices are grouped into eight categories[9] (see Table 3). Data are obtained from DataStream, and for each index we consider daily prices for the period from January 4, 1994 until September 16, 2005 if available (this corresponds to 3054 data points)[10]. Table 6 provides some descriptive statistics of the logarithmic daily percentage index returns obtained for each index. Logarithmic returns were calculated using the identity of:

$$R_{i,t} = LN(P_{i,t}/P_{i,t-1}), \tag{2.1}$$

where $R_{i,t}$ is the return on the index for period t, $P_{i,t}$ is the price of the index at the end of period t, and $P_{i,t-1}$ is the price of the index at the end of the period $t-1$.

The second data set includes information about terrorist events. We construct a database of terrorist events using publicly available information about terrorism provided by the Terrorism Research Center (TRA (2000)), the U.S. Department of State (USDS (2001)), the Constitutional Rights Foundation (CRF (2001)), the World News Map (WNM (2000)), the UK Foreign and Commonwealth Office (FCO (2005)) and the Israel Ministry of Foreign Affairs (IMFA (2005)). Data cover 77 terrorist events that occurred in 25 countries[11] for the period from January 1994 to August 2005. Though our list is subjectively determined, we select those terrorist attacks that are mentioned as significant in the aforementioned sources[12]. Each terrorist event is characterized by the date of attack, its type (armed assault, suicide bombing, bombing), the target, the place

[9] In addition to indices considered, we study the behavior of the Dow Jones Kuwait Titans 50 stock index. The data for this Islamic index covers period from January 1, 1997 till September 16, 2005.

[10] For FTSE Global Banks and FTSE Global Financials, data are available from January 2, 1996; for MSCI Europe Insurance and MSCI Europe Airlines, from January 2, 1995; for FTSE Eurozone Bond Index from May 1, 1998; and for FTSE US Bond Index, from December 31, 1999. As to S&P500 Index, the data on 12-19 September 2001 were not available because the stock market was closed due to the 9/11 attacks.

[11] Argentina, Austria, Chile, Colombia, Egypt, France, Germany, Greece, India, Indonesia, Ireland, Israel, Japan, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Russia, Spain, Thailand, Turkey, UK and USA.

[12] According to the United States Department of State, an international incident is judged *significant* if it results in loss of life or serious injury to persons, major property damage (more than $10,000$ USD), and/or is an act or attempt that could reasonably be expected to create the conditions noted (USN (2004)).

of occurrence and the number of people injured, killed or kidnapped (see Table 1).

When comparing the impact of terrorist attacks with the effect of other extreme events, we consider 4 financial crashes and 19 natural catastrophes that happened during the 11-year period considered. The financial crashes are the Mexican peso crisis in 1994, the mini-crash due to the Asian financial crisis in 1997, the Russian financial crisis in 1998 and the crisis in Argentina in 2001. The natural catastrophes include earthquakes, storms, floods, cyclones, typhoons, tornadoes, tsunami and hurricanes. Table 5 lists 10 natural disasters (from the total of 19 events under consideration) that were the most costly for the insurance industry since 1994 to 2005.

2.4.3 Methodology

In general, the most commonly used way to study the impact of events is by means of the event-study methodology (see papers by Fama, Fisher, and Jensen (1969), Brown and Warner (1980), Brown and Warner (1985), Lyon, Barber, and Tsai (1999), Chen and Siembs (2004), Abadie and Gardeazabal (2003), Dravid (1987), Pound and Zeckhauser (1990)). This methodology, however, imposes restrictive requirements on the behavior of returns on indices. In this paper, we go beyond this traditional tool and implement two other approaches. These are the non-parametric conditional density estimation approach and the filtered GARCH-EVT method. With the former, we gain the flexibility of having no assumptions about the parametric form of the data. With the latter, we account for volatility background, possible dependence between returns and the fat tail nature of their distribution. Note that in the GARCH-EVT approach we are able to check the abnormality of event-day returns only.

The Event Study Approach

We use the event study methodology to measure the magnitude of the effect of considered extreme events on the behavior of stock, bond and commodity markets. When implementing this methodology we test the hypothesis regarding the abnormality of markets' returns due to specific events. This methodology is based on the efficient market hypothesis, which states that stock prices adjust to new information[13] (see Fama, Fisher, and Jensen (1969), MacKinlay (1997)). We compute abnormal returns on the indices using a mean-adjusted return approach (Brown and Warner (1980)). This approach assumes computation of the event-day abnormal return on the index in the following way:

$$AR_t = R_t - \overline{R}, \tag{2.2}$$

where AR_t is the excess return for index at time t, R_t is the return on index at the time of event t, and $\overline{R}$ is the average return on the index taken over the interval of 20 days in the estimation window $t \in [-11; -30]$:

$$\overline{R} = \frac{1}{20} \sum_{t=-11}^{t=-30} R_t. \tag{2.3}$$

We also look at the cumulative abnormal returns (CARs) over the interval of 6 days in the post-event window. The CAR corresponding to an event that happening at time t (j=0) is computed as

$$CAR_t = \sum_{j=0}^{j=5} AR_j. \tag{2.4}$$

In contrast to event-day abnormal returns, which show the immediate investors' reaction on the terrorist event, the 6-day CARs provide an indication of the market response to the event 6 days following the attack. Usually, in event studies, the values of CARs is of more interest than the values of ARs. This is because significant negative CARs would reveal that an event had a strong impact on the markets, and insignificant negative CARs would indicate the markets' resilience to this event and their ability to recover quickly. We test the statistical significance of abnormal and cumulative abnormal returns using the test statistics described by Brown and Warner (1985).

[13]In other words, securities of a firm reflect all available information about the firm's current and future profit potential. If any information resulting from unexpected event is believed to effect a firm's current and future earnings, its security price changes as soon as the market learns about the event. To examine whether an event had any impact on the firm's values, event day abnormal returns (ARs) and cumulative abnormal returns (CARs) are measured and their statistical significance is tested.

Non-Parametric Conditional Distribution Approach

In general, non-parametric estimation is a statistical method that allows a functional form of a fit to data to be obtained without imposing any parametric assumptions. For example, a Kernel estimation of an economic model $y = M(x) + u$, requires no specification of a regression function $M(x) = E(y|x)$ and the distribution of error terms. This way, non-parametric estimation lets the data speak for themselves and overcomes a disadvantage of parametric econometrics when inconsistency between data and a particular parametric specification would result in non-robustness. At the same time, this gain in flexibility of approach is not without consequences, as non-parametric modelling has to deal with, for example, a selection of a bandwidth and a type of Kernel function. It is not the intention of this work to give a comprehensive overview on the foundations of non-parametric methodology. Rather, we want to describe an application of this powerful tool to study the impact of terrorism on different financial markets. This application can be seen as an alternative way to study the impact compared, for example, with event methodology. Note that when an impact of some event is analyzed by implementing the event study approach, the statistical significance of the effect of this event is checked by means of some test statistics. The latter, in turn, imposes some restrictions, since test statistics require some distributional assumptions with respect to the abnormal returns or cumulative abnormal returns (CARs) which have to be satisfied.

We apply a local polynomial regression (LPR) to time series data to get a non-parametric conditional distribution of stock, bond and commodity index returns. We do not compute any test statistics to check the significance of negative abnormal and/or extreme movements in the market due to terrorism. Instead, for each index and terrorist event, the value of conditional probability of a return - which is less than or equal to the one empirically observed on the day of event - is analyzed. The *abnormality* in the return corresponds to conditional probability in the interval $(0.05; 0.10]$. Where this probability is 5% or less, the return is interpreted as *extreme*. This is our subjective approach to distinguishing between extreme and abnormal movements. We assume that a terrorist attack has an impact on the index if it leads to negative abnormal and/or extreme event-day returns. Since we are interested in knowing not only the immediate reaction of the market to the event, but also the market response over some interval of time in a post-event window, we estimate a non-parametric conditional distribution of non-overlapping 6-day CARs. We make our inference about the impact of terrorist attacks in the aftermath of the event by looking at 6-day CARs in a way similar to the one described for returns. A brief description of the non-parametric estimation offered in this paper is provided below[14].

Let us consider a conditional distribution function $\pi(z|x) \equiv P(Z_i \leq z|X_i = x)$. Since we work in the time series context, X_i is a vector of lagged values of Z_i that are returns on a index. If we assume $Y_i = I(Z_i \leq z)$ then $E(Y_i \mid X_i = x) = \pi(z|x)$, consequently, the problem of estimation may be viewed as regression of Y_i on X_i.

Keeping this in mind and applying a local polynomial fitting to our time series data of index returns R_i, we minimize the following expression:

$$\sum_{i=1}^{n}(Y_i - \beta_0 - \beta_1(X_i - x_0))^2 K_h(X_i - x_0), \tag{2.5}$$

where $Y_i = \mathrm{I}(R_i \leq r_t)$ with r_t standing for empirically observed (realization) return on the day of terrorist attack t, $i = (1, \ldots, n)$, n is a sample size and n=200; $X_i = R_{i-1}$, $x_0 = r_{t-1}$; h is a bandwidth; K_h is a Kernel function.

Figure 1 provides a graphical illustration of the idea of non-parametric estimation implemented in this work. We take the 9/11 attacks as an example and build the conditional cumulative distribution function of returns on FTSE All World when conditioning is done on the return on 10th of September 2001, a day before the 9/11 attacks. The conditional cumulative probability of the return on FTSE All World, which is less than or equal to that on September 11, 2001 is found to be 0.037. Since this value is less than 0.05, we conclude that this terrorist event has an extreme event-day effect on this index.

We use a normal reference bandwidth selector (Fan and Yao (2003)), which defines an optimal bandwidth $\hat{h}_{opt,n}$ for the Epanechnikov kernel as $2.34\sigma_s n^{-1/5}$, where σ_s is a standard deviation of a sample. Implementation of this model leads to point estimates $\widehat{\beta}_0$

[14]In general, LPR is introduced into the statistical literature by Stone (1977) and Cleveland (1979). Its statistical properties are studied in papers by Tsybakov (1986), Fan (1993), Fan and Gijbels (1992), Ruppert and Wand (1994) and many others. As to estimation of non-parametric conditional density and distribution,the papers by Hall, Wolff, and Yao (1999), Linton, Chen, and Robinson (2001) and the book by Fan and Yao (2003) are useful sources on the subject.

and $\widehat{\beta}_1$:

$$\widehat{\beta} = (\mathbf{X}'W\mathbf{X})^{-1}\mathbf{X}'WY, \tag{2.6}$$

where W is a diagonal matrix, which i element is $K_h(X_i - x_0)$, and $\mathbf{X}$ is a design matrix with a first column of ones. Obtained this way, a point estimate $\widehat{\beta}_0$ corresponds to a conditional probability of return on index, which is less than or equal to that empirically observed on the day of the event (a terrorist attack) and when conditioning is done on the value of return on the previous day. The same logic applies to a sample of 200 non-overlapping 6-day CARs. The value of the CAR on which conditioning is done is computed as

$$CAR_{t-1} = \sum_{j=-1}^{j=-6} AR_j. \tag{2.7}$$

We also implement a non-parametric estimation when conditioning is implemented on the average of the returns $\bar{R}$. We believe that this approach improves the inference since the average return reflects normal market conditions better than just one return on the day before the attack.

GARCH Filter with an Extreme Value Theory Approach

When studying the impact of extreme events on financial market behavior, one can compare the event-day return on the index with the value at risk (VaR) predicted for this day and computed for different levels of significance. In the case of terrorist attacks, if the return on the day of terrorist event is lower than the computed value of VAR then we may conclude that a considered terrorist attack had an impact on the index. This method of studying the impact of events relates to the tail estimation of financial time series and requires a well-chosen way to compute the VaR that from a statistical point of view has a good predictive performance (a good fit model). In their recent paper, Kuester, Mittnik, and Paolella (2006) give an extensive and detailed overview and comparison of alternative strategies to predict VaR. They implement their study using the NASDAQ Composite Index and show that the hybrid method that combines a heavy-tailed generalized autoregressive conditionally heteroscedastic (GARCH) filter with an extreme value theory (EVT) approach performs better than other methods.

The GARCH method works as follows. First, a time-varying volatility model is applied to the time series of returns. We assume the following dynamics of returns:

$$X_t = \mu_t + \sigma_t Z_t, \tag{2.8}$$

where X_t is a strictly stationary time series representing daily observations of negative log returns on index, innovations Z_t are white noise process and have a marginal distribution function $F_Z(z)$. We assume the Gaussian distribution for innovations.

We assume that μ_t and σ_t are measurable with respect to $\Im_{t-1}$, the information about the return process available up to time $t-1$. Similar to the paper by McNeil and Frey (2000), we use the parsimonious but effective AR(1) model for the dynamics of the conditional mean:

$$\mu_t = \varphi X_{t-1} \tag{2.9}$$

and GARCH(1,1) process for the conditional volatility:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{2.10}$$

where $\alpha_0 > 0$, $\alpha_1 > 0$ and $\beta > 0$, $\epsilon_t = X_t - \mu_t$ and $\alpha_1 + \beta < 1$.

For each terrorist attack, we take a sample from 200 to 2500 past return observations[15] (starting one day before the attack) and fit this model to the data by means of the pseudo-maximum likelihood method to get the estimates of parameters $\widehat{\theta} = (\widehat{\varphi}, \widehat{\alpha}_0, \widehat{\alpha}_1, \widehat{\beta})$. Estimates of the conditional mean series $(\widehat{\mu}_{t-n+1}, ..., \widehat{\mu}_t)$ and the conditional standard deviation series $(\widehat{\sigma}_{t-n+1}, ..., \widehat{\sigma}_t)$ are obtained recursively from (2.9) and (2.10) using reasonable starting values. When correctly specified and with a good fit, this model allows us to obtain filtered residuals

$$(z_{t-n+1}, ..., z_t) = \Big(\frac{x_{t-n+1} - \widehat{\mu}_{t-n+1}}{\widehat{\sigma}_{t-n+1}}, ..., \frac{x_t - \widehat{\mu}_t}{\widehat{\sigma}_t}\Big) \tag{2.11}$$

that are approximately iid, which is an important requirement for the EVT approach applied below. Finally, the estimates of the conditional mean and variance for day $t+1$

[15] We vary the sample size to get good backtesting results.

are given by $\widehat{\mu}_{t+1} = \widehat{\varphi} x_t$ and $\widehat{\sigma}^2_{t+1} = \widehat{\alpha}_0 + \widehat{\alpha}_1 \widehat{\epsilon}^2_t + \widehat{\beta}\widehat{\sigma}^2_t$, where $\widehat{\epsilon}_t = x_t - \widehat{\mu}_t$.

We estimate the tail of the standardized residuals by means of the EVT, namely, by applying the Peak-Over-Threshold methodology (POT). The latter approach focuses on the distribution of excess returns over some threshold and applies a key result, due to Pickands (1975), that the Generalized Pareto Distribution (GPD) is the limit distribution of scaled excesses of iid random variables over high threshold. This distribution has the following cdf for $\xi \neq 0$:

$$H_{\xi,\beta}(y) = 1 - \left[1 + \frac{\xi y}{\beta}\right]^{-1/\xi}, \tag{2.12}$$

where $\beta > 0$, $y \geq 0$ when $\xi \geq 0$ and $0 \leq y \leq -\beta/\xi$ and when $\xi < 0$. When $\xi = 0$, the expression in (12) gets the form of $H_{\xi,\beta}(y) = 1 - \exp\left(\frac{-y}{\beta}\right)$. When implementing the EVT estimation, we first order the residuals $z_{(1)}, ..., z_{(n)}$ and fit the distribution in (2.12) to the data $(z_{(1)} - z_{(k+1)}, ..., z_{(k)} - z_{(k+1)})$, the excess amounts over the threshold $z_{(k+1)}$ with k standing for the number of data in the tail. The quantile estimate $\widehat{z}_q$ for $q > 1 - k/n$ is

$$\widehat{z}_q = z_{(k+1)} + \frac{\widehat{\beta}_k}{\widehat{\xi}_k}\Big(\Big(\frac{1-q}{k/n}\Big)^{-\widehat{\xi}_k} - 1\Big). \tag{2.13}$$

Finally, the estimate of the VaR is computed. If we denote the marginal distribution of X_t as $F_X(x)$ and let $F_{X_{t+1}+...+X_{t+k}|\Im_t(x)}$ be the predictive distribution of returns over the next k days then the quantile of the latter distribution is given by

$$VaR^t_q = x^t_q(k) = \inf\{x \in R : F_{X_{t+1}+...+X_{t+k}|\Im_t(x)} \geq q\}. \tag{2.14}$$

Because $F_{X_{t+1}|\Im}(x) = P\{\sigma_{t+1} Z_{t+1} + \mu_{t+1} \leq x \mid \Im_t\} = F_Z((x - \mu_{t+1})/\sigma_{t+1})$ we can compute VaR as

$$\widehat{VaR}^t_q = \widehat{x}^t_q = \widehat{\mu}_{t+1} + \widehat{\sigma}_{t+1}\widehat{z}_q, \tag{2.15}$$

where $\widehat{z}_q$ is the upper qth quantile of the marginal distribution of Z_t obtained using (2.13). Computed this way, the VaR accounts for the volatility background and fat-tail nature of the distribution of index returns. These are two important stylized facts of most financial return series.

To evaluate the predictive power of the above approach, we implement a backtesting procedure described in McNeil and Frey (2000). We update the AR(1)-GARCH(1,1) model parameters for 500 moving windows and produce 500 one-step-ahead forecasts of $\widehat{VaR}_q^t = \widehat{x}_q^t$ that are subsequently compared with actually observed values of returns x_{t+1} for $q \in \{0.90, 0.95, 0.99\}$. We implement this procedure when studying the impact of every terrorist attack. A violation is said to occur when $x_{t+1} > \widehat{x}_q^t$. Finally, given that the total number of violations is binomially distributed, we test the hypothesis that the model estimates the conditional quantiles correctly.

2.4.4 Empirical Results

Summary

- *Hypothesis 1: Terrorist attacks do not have a significant effect on global, European, American and Swiss stock markets.*

  The results obtained reject this hypothesis and show a significant negative impact of terrorist events on the above mentioned markets: according to the event-study, 55 out of 77 terrorist attacks (56 in the non-parametric case, 45 according to the GARCH-EVT method) have a significant negative impact on the behavior of at least one of these markets. The Swiss market is affected by the highest number of attacks while the American market is affected by the lowest number of events. The reasons for a strong reaction of the Swiss market to terrorist events can relate to several factors. This may be because this index is comprised of fewer companies than the S&P500 (less broad index). In addition, the SMI's sensitivity may be explained by the fact that this index includes stocks of companies that operate internationally and that are potentially more sensitive to extreme events due to the nature of their business. The results obtained for the S&P500 are quite reasonable given the fact that only 4 out of 77 terrorist attacks we consider took place in the U.S. In addition, the resilience of the American market to terrorist attacks can be explained by the stable banking/financial sector in the U.S., which provides adequate liquidity to promote market stability.

- *Hypothesis 2: Terrorist attacks do not have a significant effect on such industry indices as insurance, travel/pleasure, airline, oil and gas, financials and banking.*

  The empirical evidence rejects this hypothesis and suggests a significant negative impact of terrorist events on the above-mentioned industries. According to the event-study 61 terrorist attacks (55 in the non-parametric case, 41 according to the GARCH-EVT method) lead to significant negative return movements in at least one industry index. Insurance and airline sectors exhibit the highest susceptibility to these events (the MSCI Europe Insurance is affected by the highest number of attacks), while the banking sector is affected the least. These results are quite intuitive. Terrorist attacks often lead to fatalities and significant damage to property that explains a high sensitivity of the insurance sector to terrorist risk. The results support the conclusions of several studies (see Raby (2003), Bonturi, Koen, and Lenain (2002), Abadie and Gardeazabal (2003), Darkos and Kutan (2001), Enders, Sandler, and Parise (1992)) that identify the airline, travel, tourism and insurance sectors as those that are particularly sensitive to terrorist events. With respect to the lower level of impact on the banking sector, it is possible that banks' operations are not directly related to the businesses that suffered from the terrorist events. Finally, the oil/gas stock industry shows both significant negative and positive return response[16]. This effect is observed at both global and European levels. The negative reaction of these indices is observed more often than the positive and can be explained by a fear of possible economic slowdown and a decrease in consumer confidence. This especially relates to the transportation sector, for example, by a drop in air travel. In turn, this leads to a lower oil demand and a decrease in oil prices[17]. At the same time, a positive effect on oil prices is often related to the place of attack (whether it can cause a danger to oil production and transportation[18])

[16] The same reaction is observed at an individual oil company level: we analyze the impact of terrorist events on the stock returns of Exxon.

[17] For example, '...oil fell more than 2 USD to 74 USD a barrel and US petrol futures tumbled to a seven-week low on Thursday after Britain said it had thwarted a plot to blow up trans-Atlantic aircraft flights and investors recalled the slump in fuel demand that followed 9/11. Oil consumption would again be hard hit if travelers turn away from flights and consumer confidence takes a knock. Jet fuel prices in particular moved down sharply in the weeks after September 11, 2001 and after the SARS outbreak in 2003'. New Zealand Herald (2006).

[18] 'If someone can fly planes into buildings, they can fly planes into a production facility,' Emerson said. 'So it's not irrational that there's this fear premium in the market', Washington Post (2004).

and the oil market conditions at the time of event (if an attack occurs when the market is tight because of increasing global demand). Importantly, in this study we analyze the impact of terrorist events on returns on the event-day and in the post-event window of 6 days after the attacks. Therefore, conclusions drawn from our investigation, relate to the markets' short-term reaction only.

- *Hypothesis 3: Terrorist attacks do not have a significant effect on such industry indices as defense and pharmaceutical/biotechnology.*

  The analysis of the impact does not support this hypothesis and shows a significant positive reaction of these indices across all methodologies. Significant positive impact on both sectors is observed for the bombings in Pakistan in 2002 (extreme return movements). Significant positive impact on the pharma/biotech industry is found for the bombings in Indonesia/Bali in 2002 and the armed assault in Colombia in 2005 (abnormal return movements). Significant positive impact on the defense industry is identified for the bombing in Oklahoma City in 1995 (abnormal return movements). When the post-event impact is examined over a longer time window (11-day CARs and 30-day CARs), we find a significant positive response to such terrorist attacks as that of 9/11, the bombings in Madrid and Egypt in 2004 and in London in 2005. The possibility of a positive reaction by the defense industry to terrorist events is also suggested in the OECD report (Bonturi, Koen, and Lenain (2002)). One of the explanations of this result that we may suggest is that terrorist events may induce the increase in government expenditure on defense and on research in the pharma/biotech area in relation to preventive actions against possible chemical or biological terrorist attacks[19].

- *Hypothesis 4: Terrorist attacks do not have a significant effect on the commodity and gold markets.*

  The analysis of the impact rejects this hypothesis and shows a significant positive

[19]'The tragic events in the United States on 11 September 2001 show that it can no longer be assumed that the deliberate causing of mass civilian fatalities is impossible in the developed world. It will thus be necessary to look again at the problems caused by the proliferation of nuclear, chemical and biological weapons of mass destruction. Of these, biological weapons pose by far the greatest threat, because they can be as lethal as nuclear weapons and are easier to obtain. In fact, a crude attack with biological weapons would probably be easier to plan and execute than was the attack on the World Trade Center' (Fraser and Dando (2001).)

reaction of the commodity and gold market' returns to terrorist events. In addition, a significant negative reaction is observed for some events. The gold market shows more negative sensitivity to terrorist events compared to the commodity and bond markets. Given that gold is usually considered to be a 'safe-haven' asset, these empirical results remain difficult to explain.

Commodity and gold markets respond positively to some terrorist events (the 9/11 attacks) and show no significant reaction to other (the bombings in Egypt in 2004). Finally, some events, as for example the London bombings in 2005, cause significant negative return movements in the commodity index and have no effect on the gold index. Such behavior implies that investing in the commodity/gold markets as a hedging strategy against terrorism risk may not always work. This is because, with terrorist events, these markets can react negatively.

- *Hypothesis 5: Terrorist attacks do not have a significant effect on the bond market.*

  The analysis of the impact rejects this hypothesis and shows a significant positive reaction of the bond market' returns to terrorist events. In addition, a significant negative reaction is observed for some events. The negative impact of some attacks is mostly observed on the event-day only. Compared to other bond indices, the Global Government Bond Index experiences significant positive return movements more often than negative return movements. The FTSE U.S. Government Bond Index displays the lowest level of impact, both positive and negative.

- *Hypothesis 6: Terrorist attacks do not have a significant effect on financial markets on both the event-day and in the post-event window.*

  The empirical results reject this hypothesis and show that terrorist events lead to a significant response in financial market returns on both the event-day and in the post-event window.

  In most of the cases, the event-day stock return movements associated with attacks are extreme and the strength of the impact declines in the post-event period.

  In relation to commodity markets, significant return movements in the gold index both negative and positive are extreme and often observed in the post event period.

In contrast, a negative reaction by the Goldman Sachs Commodity Index is found in all periods and a positive response is mostly observed on the event-day.

Among bond indices, the global and European bond markets react negatively in all periods, while return movements are more often extreme than abnormal. Unlike these two markets, the U.S. bond market responds positively to terrorist events mostly on the event-day and associated returns have an abnormal character.

All financial markets perceive terrorist attacks as unusual events. While some of them see the effects of these events as occurring mostly on the event-day only (the U.S. bond market), some markets take a longer time to evaluate the impact and reveal their reaction mostly in the post-event period (the gold market). Finally, some markets (stocks, commodities, global and European bonds) react to terrorism either on the event day or in the post-event window or both.

- *Hypothesis 7: Terrorist attacks do not have an effect on financial markets which is similar to that of natural catastrophes and financial crashes.*

  The results obtained partly support this hypothesis. There are both similarities and differences between the impact of terrorist events on financial markets and the effect of financial crashes and natural disasters. While the European and Swiss markets show high susceptibility to terrorist attacks and natural catastrophes, their response to financial crashes is less negative. At the industry level, the insurance and airline sectors show a negative sensitivity to all types of the extreme events. Financial crashes demonstrate a strong negative impact on the banking and financials sectors. This is in contrast to the effect of terrorist attacks and natural catastrophes that do not cause a strong negative response in the sectors mentioned above. Similar to terrorist events, there are observed both a positive and negative impact of natural disasters and financial crashes on such industries as oil/gas and pharma/biotech.

  The event-day negative returns associated with financial crashes and terrorist events are extreme. The sensitivity of stock markets to these events declines in the post-event window. In contrast, natural disasters are associated with extreme return movements more in the days following the events. This result may be because markets need more time to evaluate the long-term consequences of natural disasters on markets' returns compared to the two other types of events.

Both terrorist events and natural disasters cause positive and negative return movements in the commodity/gold and bond markets. Among the latter markets, the U.S. bond market shows the least impact from all extreme events considered. With respect to the impact of financial crashes, our empirical findings confirm a traditional perception of the commodity and bond markets as those providing 'safe-haven' investment opportunities in times of crises. This is because with financial crashes these markets react positively.

Summary of the impact of different extreme events on the financial markets is given in the Table 7.

The Impact of Terrorist Attacks on Stock Markets

- **The Event-Study Approach**

  Tables 8-9 show the results of the event study. Analysis of the response of four stock indices - FTSE All World, MSCI Europe, S&P500 and SMI to terrorist attacks shows that 22 out of 77 attacks have no impact on any of these stock markets. Among these events are not only local attacks that are characterized by very little or no damage to property and people as for example, the bombing in Israel on May 27, 2001, but also such events as the attacks in Argentina on July 18, 1994 that are considered as one of the worst in terms of fatalities and the bombings in Sri Lanka/Colombo on July 24, 2001 that are reported among the worst in terms of insured property loss during the period 1970-2001 (OECD (2005)). In other words, the impact of attacks on stock markets is not necessarily in direct relation to their magnitude in terms of insured losses and fatalities.

  Our investigation shows that 55 out of 77 terrorist events have a significant negative impact on at least one stock index. FTSE All World and Swiss indices are affected by the highest number of events while the S&P500 Index is affected by the least number of attacks. We also check the reaction of Islamic stock markets by looking at the

DJ Kuwait Titans 50's response to terrorist events. This index experiences both a significant negative and a significant positive impact of terrorist attacks and natural disasters. In contrast, financial crashes cause only a significant negative response in this index returns (see Tables 28-29). This index is affected in a negative way by a lower number of attacks than are other market indices. A negative effect of attacks is more pronounced compared to a positive impact. While a positive reaction is mostly observed on the event day, significant negative return movements are found more often in the post-event window. These results indicate that the Kuwait stock market perceives terrorist events as negative news.

The 9/11 attacks as well as the suicide bombing in Israel on June 19, 2002 and bombings in Madrid 2004, in Egypt 2004 and in the UK 2005 are good examples of events that have a negative impact on stock markets at both global and local levels. We find that the 9/11 terrorist attacks have a significant negative effect on global, European, American and Swiss stock markets both on the event day and in the post-event window. The S&P500 shows the strongest negative reaction in the post-event window compared to other indices, which reflects a prolonged negative effect of the 9/11 event on the American market. As to the negative impact of this event on the European market, our results find support in the empirical paper by Chen and Siembs (2004), where the authors conclude that European capital markets experience significant negative 6-day CARs due to the 9/11 attacks.

The empirical results for various industry indices show that 62 out of 77 terrorist events have a significant negative effect on at least one of them. The insurance sector is affected by the highest number of events while the banking and oil/gas industries are affected by the lowest number of attacks. Within the insurance sector, MSCI Europe Insurance experiences the most negative impact and is closely followed by FTSE All World Non-Life Insurance. These results are quite intuitive since terrorist attacks often lead to fatalities and significant damage to property that explains a high sensitivity of the insurance sector to terrorism risk. Unlike the insurance industry, the banking sector is affected by the least number of attacks. It is possible that banking operations are not directly related to the businesses that suffered from the terrorist events.

We find evidence of the significant negative impact of terrorist attacks on the FTSE All World Travel/Pleasure and MSCI Europe Airlines. For both, almost half of the attacks considered lead to a significant negative reaction. While the FTSE All World Travel/Pleasure is affected more often in the post-event period, for MSCI Europe Airlines this type of impact is observed less frequently. In addition, when characterizing the impact on the airline index, we see that more than half the terrorist events (out of 31 that have an impact) cause significant negative return movements on both the event-day and the post-event window, reflecting a high susceptibility of this sector to terrorism risk. These results support conclusions of several studies (see Raby (2003), Bonturi, Koen, and Lenain (2002), Abadie and Gardeazabal (2003), Darkos and Kutan (2001), Enders, Sandler, and Parise (1992)) that identify airline, travel, tourism and insurance sectors as those that are particularly sensitive to terrorist events.

Analysis of the ARs and 6-day CARs shows evidence of significant negative (see Tables 8-9) as well as positive (see Table 10) impact of terrorist attacks on the aero/defense and pharma/biotech industries. Although the latter index is affected less often, however, the strength of the impact is higher compared to aero/defense. Among the events that have a significant positive impact both, on the event day and in the post-event window are bombings in Oklahoma City in 1995 (FTSE All World Aero/Defense) and in Pakistan in May 2002 (FTSE All World Aero/Defense and FTSE All World Pharma/Biotech) and the armed assault in Colombia in April 2004 (FTSE All World Pharma/Biotech). If we consider four better known terrorist events such as the 9/11 attacks, the bombings in Madrid and in Egypt in 2004 and the bombing in London in 2005, we see that the reaction of these indices is negative except for attacks in London in 2005, which have no effect on the aero/defence industry and lead to a significant positive post-event window reaction in the pharma/biotech industry. However, when the post-event impact is examined over longer time horizons (11-day and 30-day CARs instead of 6-days), the results are different. For the FTSE All World Aero/Defense index, we get statistically significant positive 30-day

CARs that are associated with all the above-mentioned events. As to FTSE All World Pharma/Biotech sector, only the bombings in Madrid and in London lead to statistically significant positive 30-day CARs[20].

The aero/defense and pharma/biotech sectors are not the only industries that experienced both a positive and a negative impact of terrorist attacks. An analysis of the reaction of the oil/gas sector also reveals these two types of impact (see Tables 8-9, 11). This mixed effect is observed at both global and European levels. Among events that lead to significant positive return movements both on the event-day and in the post-event window are the suicide bombings in Russia in December 2002. Unlike in the aero/defense and pharma/biotech industries, some terrorist events first cause a significant positive event-day response in the oil/gas industry and then lead to a significant negative impact in the post event period. This behavior is observed for the 9/11 attacks[21], the suicide bombings in Israel in 2002 (FTSE All World Oil/Gas, FTSE Europe Oil/Gas), the kidnapping in Indonesia in 1996 and the bombings in Russia in March 1999 (FTSE All World Oil/Gas). This implies that these markets perceive these attacks as having a negative impact but this impact is not initially distinguished on the event-days. Interestingly, the bombings in Madrid in 2004 cause a significant negative event-day abnormal return movements followed by a significant positive response in the post event window (FTSE Europe Oil/Gas). In this case, the immediate negative reaction of the market may possibly be explained by the fact that these bombings are targeted at the transportation system. As new information is processed with respect to the long-term effect, the market reveals no fear. Note that although the aero/defense, pharma/biotech and oil/gas industries show both, positive and negative response to terrorist attacks, the latter type of impact is more than twice as frequent. In terms of the strength of the impact, for all indices considered, terrorist events more often cause negative ARs and CARs that are statistically significant at 0.01 and 0.05 levels rather than at 0.10 level.

[20]Note that we checked the 11-day CARs and 30-day CARs for other indices such as travel/pleasure, banking and financials. We did not find any evidence suggesting a significant positive impact of attacks on these indices, implying that only aero/defense and pharma/biotech sectors exhibited a positive reaction to attacks over longer time horizons.

[21]It is worth noting that after the September 11, 2001 attacks on New York City, oil prices fell by almost 40%.

- **The Non-Parametric Approach**

  The findings of our non-parametric estimation are summarized in Tables 10-11, 16-17. The results in relation to the event-day impact of attacks are obtained when conditioning is implemented on the average return.[22] Similar to the findings of the event-study approach, the impact of attacks on the stock markets is not necessarily in a direct relation with their magnitude in terms of insured losses and fatalities. The list of terrorist events that do not show any impact is similar to the one suggested by the event-study approach. Our investigation shows that 56 out of 77 terrorist events have an impact on at least one stock market under consideration. The Swiss and European indices are affected by the highest number of events, while the S&P500 Index is affected by the least number of attacks. These results are similar to those revealed in the event study. The non-parametric approach suggests similar results with respect to the impact of extreme events on the Islamic Dow Jones Kuwait Titans 50 to those found in the event-study approach (see Tables 28-29).

  For all indices in category I (FTSE All World, MSCI Europe, S&P500 and SMI), terrorist attacks more often lead to an event-day negative response and less often to a prolonged negative reaction. Similar to the results of the event study approach, all stock indices experience extreme event-day negative return movements more often than abnormal return movements (see Table 16). The strength of the impact declines in the post-event period. This result also applies to the industry indices with the exception of airline, aero/defense and pharma/biotech sectors.

  We find that 55 terrorist attacks make significant negative impact on at least one industry index. This result reflects the more conservative nature of the non-parametric approach compared to the event-study methodology. The latter suggests that a larger number of events cause a negative market response. This result can be due

[22] There are several reasons for this. First, as we mentioned before, the return on the day before the attack might not represent normal market conditions as accurately as the average return. Secondly, analysis of the data shows that there are quite a few terrorist attacks that have low even-day impact when conditioning is implemented on the previous return, no impact in the post-event window and no event-day impact when conditioning is performed on average returns. In addition, these attacks are such that their magnitude or place of occurrence suggest that no impact on the given stock index is a reasonable result. Examples of such events for MSCI Europe are the bombings in Sri-Lanka on July 7, 2001 and in Israel on August 4, 2002 and the armed assault in Pakistan on August 5, 2002.

to restrictive assumptions imposed by test statistics used in the event studies. At the same time, the findings across different industries are quite similar among these methodologies. The insurance and airline sectors show high sensitivity to terrorism, while the FTSE Europe Oil/Gas, followed by the banking sector is affected by the lowest number of attacks. Similar to the findings of the event-study approach, the aero/defense, pharma/biotech and oil/gas sectors exhibit both positive and negative abnormal return movements associated with terrorist events. Events that cause positive reaction are similar to those identified in the event-study (see Tables 26-27).

- **The GARCH Filter with EVT Approach**

  The results provided by this method describe the event-day impact of terrorist attacks only[23] (see Tables 22-23). 45 out of 77 terrorist events have a significant negative impact on at least one stock index. More than half of these events cause extreme rather than abnormal event-day returns. The Swiss stock index is the most often affected, while American and European markets exhibit the lowest level of the event-day impact.

  At the industry level, 41 out of 77 attacks lead to significant negative event-day return movements in at least one industry index. The insurance and airline industries exhibit the highest susceptibility to terrorism (the MSCI Europe Insurance is affected by the highest number of attacks). The oil and pharma/biotech sectors exhibit the least negative event-day impact. The GARCH-EVT method suggests a greater negative impact on the aero/defense and a lesser negative impact on the pharma/biotech compared to the other two methods. At the same time, it also displays the presence of significant positive event-day return movements in these two sectors, a result which is similar to the findings of the other two methods.

  Some terrorist events, namely the bombing in Sri-Lanka/Colombo in 1996, the armed assault in the US in February 1997 and the kidnapping in Indonesia in February 1997 are identified as those causing a significant positive impact on the pharma/biotech sector. The impact of these events, however, is not identified by other methods. With the exception of the above-mentioned events, the list of attacks that lead to

[23]See section 4.4.5 that explains limitations of the GARCH-EVT method.

the positive event-day impact is similar across all methodologies (see Tables 26-27). Finally, in respect to the oil/gas sector, the GARCH-EVT approach shows a significant negative event-day impact of a smaller number of attacks and a significant positive impact of a greater number of attacks compared to other two methods.

The Impact of Terrorism Events on the Commodity and Bond Markets

The results of our empirical analysis across all methodologies are presented in Tables 12-15, 18-21. The gold market experiences significant negative movements associated with terrorist events more often than positive movements. This is in contrast to the behavior of the commodity and bond markets, which show significant positive reaction to attacks more frequently than significant negative reaction. While a negative reaction by the gold market is more often observed in the post-event period, other markets respond negatively only on the event day. The latter case implies that the commodity and bond markets show less negative sensitivity to terrorism than does the gold market. We do not find any pattern that would relate the negative reaction of the gold index' returns to the country where terrorist event takes place.

The gold and commodity indices exhibit a significant positive response to the 9/11 attacks while neither a positive nor a negative significant reaction is associated with the bombings in Egypt in 2004. The commodity market exhibits significant positive return movements associated with the bombings in Madrid in 2004 and significant negative return movements associated with the bombings in London in 2005. In contrast, the gold market shows no significant response to either of these events. Note that, in general, commodities/gold and bonds are considered 'safe-haven' assets that benefits from unsettling world news. At the same time, the above-mentioned results show the presence of a negative response by these markets to terrorist events. Such empirical evidence remains puzzling.

The Global Government Bond index experiences significant positive and negative return movements. The former more often than the latter. The FTSE U.S. Government Bond index displays the lowest level of reaction, positive or negative.

According to all methodologies, the 9/11 attacks and the bombings in Turkey in 2004 cause significant positive response in the global bond index. The suicide bombings in London in 2005 have a positive impact on the U.S. bond market, according to the non-parametric approach. Other two methodologies do not reveal any impact of this event on the bond markets. The bombings in Madrid in 2004 have a positive effect on the world bond market according to the event-study and the non-parametric approach. The bombings in Egypt in 2004 cause neither positive nor negative significant return movements in the bond markets. As with commodities and gold, investing in bonds can be a possible investment strategy to hedge against terrorism risk. However, one should be aware of the possibility of a significant negative response of these assets' returns to terrorist attacks.

Effect of Natural Hazards and Financial Crashes

Event-study assessment of the impact of natural catastrophes shows that 7 out of 19 natural disasters[24] have a negative impact on at least one of the stock markets considered. This compares to 9 events distinguished in the non-parametric approach. The GARCH-EVT methodology reveals a significant negative event-day effect of 4 natural catastrophes. Across all methodologies, the European and Swiss indices are affected by the highest number of natural hazards. A possible reason for this could be that many of the events considered occur in Europe. As to the SMI index, its negative reaction may be due to a more significant presence of insurance/reinsurance companies in the construction of this index. The American market, followed by the global market is affected by the lowest number of catastrophes. An event-day reaction of stock markets to natural catastrophes can be characterized by both extreme or abnormal negative return movements. As to a post-event window, the response is more often extreme than abnormal.

As with terrorist events, natural catastrophes cause both positive and negative return movements in the commodity/gold and bond markets. The gold index is affected by a lower number of events compared to the commodity index, implying lower sensitivity of a former market to natural disasters. The FTSE U.S. Government Bond index shows the lowest level of the impact and the European bond index demonstrates most of the impact associated with natural hazards.

[24] For the FTSE All World the list of events covers 18 natural hazards.

The results of the impact of financial crashes are as follows. While all stock indices considered show a significant negative reaction to the mini-crash due to Asian crisis of 1997, none of them experience a significant negative impact after the crises in Mexico and Argentina. Although the European and Swiss stock markets show high susceptibility to natural disasters and terrorist events, these markets demonstrate less of a negative impact after financial crashes.

At the industry level, the insurance sector and the airline industry show negative sensitivity to both financial crashes and natural disasters. Financial crashes show strong negative impact on the FTSE Global Banking and the FTSE Global Financials. This is a reasonable result given the direct connection of these industries to the financial markets. There is a positive and negative impact of financial crashes and natural disasters on such industries as the oil/gas and pharma/biotech (see Tables 8-11 and 16-17).

Event-day negative returns associated with financial crashes and terrorist attacks are mostly extreme, with the strength of the impact declining in the post-event period. By contrast, natural catastrophes cause significant return movements, which are mostly in the post-event window.

Financial crashes cause a significant positive reaction in the commodity and gold markets. While the commodity market responds positively mostly in the post-event window, the gold market responds positively both on the event-day and in the post-event period. With respect to the global bond index, financial crashes cause a significant positive event-day and a post-event window return movements across all methodologies. Financial crashes lead to no significant response in the U.S. bond index. The above-mentioned empirical findings confirm a traditional perception of the commodity and bond markets as those providing 'safe-haven' investment opportunities in times of crises.

Different Approaches: Summary

The results presented in Table 22 and 23 display respectively a negative and positive event day impact of extreme events on the financial markets. Tables 24-25 summarize the findings of our empirical work across different methodologies. Comparing methodologies, the GARCH-EVT approach shows the least number of extreme events that lead to significant negative event-day return movements in the indices considered. When it comes to a positive impact, the event-study approach often reveals a significant effect of a lesser number of events. For 5 indices out of 7 that experience both types of impact, the GARCH-EVT method shows more of the positive impact compared to other methodologies. The differences in the impact across methodologies can relate to the underlying assumptions they impose on the market returns. The GARCH-EVT approach, for example, accounts for the volatility background, dependence and fat-tail nature of the market returns. These are important characteristics of financial markets' returns that are not captured by the other two methodologies. The letter methods can overestimate or underestimate the impact of events. At the same time, the GARCH-EVT approach allows to study the event-day effect only and is more computationally intensive .

2.5 Conclusions

This study shows results regarding the global, regional, national and industrial effects of terrorist events on stock markets as well as the impact of attacks on the commodities and bonds. Also, it compares the impact of terrorist events on financial markets with the effect of natural catastrophes and financial crashes.

Around two thirds of the terrorist attacks considered lead to significant negative impact on at least one stock market under consideration. The Swiss stock market is affected by the highest number of attacks, the American stock market by the lowest.

The reasons for a strong reaction of the Swiss market to terrorist events can relate to several factors. This may be because this index is comprised of fewer companies than the S&P500 (less broad index). In addition, the SMI's sensitivity may be explained by the fact that this index includes stocks of companies that operate internationally and that are potentially more sensitive to extreme events due to the nature of their business. The results obtained for the American market are quite reasonable given the fact that only 4 out of 77 terrorist attacks we consider took place in the U.S.

The airline industry and insurance sector exhibit the highest susceptibility to terrorism, while the banking industry is the least sensitive. This is in contrast to financial crashes which demonstrate a strong negative impact on the banking sector. The analysis of the impact on the aero/defense, pharma/biotech and oil/gas sectors shows both a positive and a negative reaction. These stock markets behave similarly in case of the natural disasters and financial crashes.

The results of our study suggest several diversification strategies against terrorism risk. If concerned about this risk, investors should hold assets that can react positively to terrorist attacks or, alternatively, that have little or no negative sensitivity to this risk. In the first case, the U.S. Government bond index is the safest choice followed by such industry stocks as aero/defense and pharma/biotech. However, given that these stock markets can also exhibit a negative response, investing in these industries as a diversification strategy against terrorist attacks may not always work. In the second case, banking stock index can be a good investment. Note that, though the this stock index is less sensitive to terrorist attacks, it exhibits significant negative return movements associated with financial crashes.

In relation to terrorist attacks, investing in commodities is preferable to investment in gold. As the gold market reacts negatively more often than positively. In addition, compared to the commodity market in general, the negative impact on the gold market is more long-lasting. At the same time, the commodity market also shows a short-term negative reaction to some terrorist events. This implies that investing in the gold and commodity markets may not always provide a good hedge.

A possible way to reduce negative exposure to terrorist events would be to avoid investing in the insurance, travel and airline stock markets. Note that the insurance and airline industries shows high negative sensitivity not only to terrorist attacks but also to financial crashes and natural disasters. This implies that by keeping these stocks in their portfolio, investors may end up increasing the risk of losses in these cases where extreme events occur.

Finally, the response of financial markets to terrorist events suggests several strategies of trading derivatives. For example, investors can hold a long position in put options on the stock industry sectors that react negatively to terrorist events. Or alternatively, they can invest in call options on the bond index, where the underlying asset is the U.S.

Government bond index.

There are both, similarities and differences between the impact of terrorist events on financial markets and the effect of other extreme events. For example, the insurance and airline industries show high sensitivity to all three categories of extreme events. The banking industry shows little negative impact of natural hazards, which is similar to the impact of terrorist attacks and in contrast to the effect of financial crashes. Terrorist attacks and natural disasters cause both, positive and negative significant return movements in the commodity and bond markets. In contrast, financial crashes have a positive effect on these markets. Terrorist attacks and financial crashes cause an event-day return movements that mostly has an extreme nature, with the strength of the impact declining in the post-event period. As to natural catastrophes, the negative impact is more often observed in the post-event period, implying that markets need more time to evaluate the long-term effect of these extreme events.

Appendix A: Figures

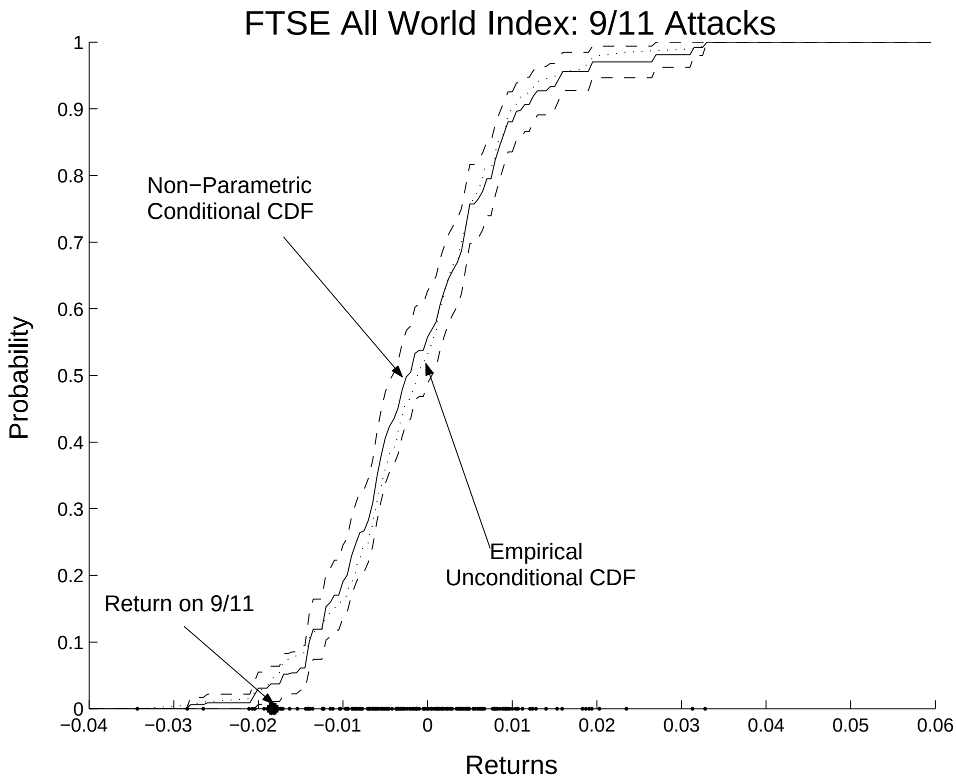


Figure 1. FTSE All World Index: 9/11 Attacks.

The figure shows a non-parametric conditional cumulative distribution function of returns on FTSE All World obtained based on 200 observations when conditioning is done on the return on 10th of September 2001, a day before the 9/11 attacks. The dotted line corresponds to an empirical unconditional cdf, dashed lines correspond to 95% confidence intervals.

Appendix B: Tables

N	*Date of attack*	*Country*	*Type of attack*	*Target*	*Killed*	*Wounded*	*Other*
1	18.07.1994	Argentina	Bombing	Personnel	86	300	
2	22.01.1995	Israel	Bombing	Transport	21	69	
3	19.04.1995	U.S.A	Bombing	Facilities	168		
4	19.11.1995	Pakistan	Suicide bombing	Government	16	60	
5	08.01.1996	Indonesia	Kidnapping	Personnel			26
6	31.01.1996	Sri Lanka/ Colombo	Bombing	Facilities	100	1500	
7	09.02.1996	UK	Bombing	Personnel	6		
8	18.02.1996	UK	Bombing	Transport	1	9	
9	25.02.1996	Israel	Suicide bombing	Transport	26	80	
10	04.03.1996	Israel	Bombing	Facilities	20	75	
11	15.06.1996	UK	Bombing	Facilities		206	
12	02.01.1997	U.S./UK	Bombing	Facilities			
13	23.02.1997	U.S.	Armed Assault	Personnel	1		
14	24.02.1997	Colombia	Kidnapping	Personnel			1
15	13.03.1997	Israel	Armed Assault	Personnel	7	30	
17	17.04.1999	UK	Bombing	Transport	2	30	
18	20.04.1999	India	Bombing	Facilities	5	47	
16	13.09.1999	Russia	Bombing	Facilities	118		
19	22.11.2000	Israel	Bombing	Transport	2	60	
20	01.01.2001	Israel	Bombing	Transport		60	
21	03.01.2001	Switzerland	Bombing	Facilities			
22	25.05.2001	Israel	Bombing	Transport		65	
23	27.05.2001	Israel	Bombing	Personnel			
24	24.07.2001	Sri Lanka/ Colombo	Bombing	Transport	12		
25	11.09.2001	U.S.A	Bombing	Facilities	3000		
26	29.10.2001	Israel	Suicide bombing	Transport	3	9	
27	29.10.2001	Russia	Suicide bombing	Military	3	1	
28	28.10.2001	Pakistan	Armed Assault	Facilities	15		
29	13.12.2001	India	Armed Assault	Government	7		
30	27.01.2002	Israel	Suicide bombing	Personnel	1	100	
31	09.03.2002	Israel	Suicide bombing	Personnel	11	52	
32	17.03.2002	Pakistan	Armed Assault	Facilities	5	46	
33	20.03.2002	Peru	Bombing	Transport	10	38	
34	10.04.2002	India	Armed Assault	Facilities	5	4	
35	10.04.2002	Israel	Suicide bombing	Personnel	8	22	

Table 1: Terrorism Events

N	*Date of attack*	*Country*	*Type of attack*	*Target*	*Killed*	*Wounded*	*Other*
36	08.05.2002	Pakistan	Bombing	Transport	12	19	
37	09.05.2002	Russia	Bombing	Personnel	42	150	
38	19.05.2002	Israel	Suicide bombing	Facilities	3	59	
39	14.06.2002	Pakistan	Bombing	Transport	11	51	
40	19.06.2002	Israel	Suicide bombing	Transport	6	43	
41	17.07.2002	Israel	Suicide bombing	Transport	5	38	
42	31.07.2002	Israel	Bombing	Facilities	9	87	
43	04.08.2002	Israel	Bombing	Transport	9	50	
44	05.08.2002	Pakistan	Armed Assault	Facilities	6	1	
45	12.10.2002	Indonesia	Bombing	Transport	202	300	
46	26.12.2002	Philippines	Armed Assault	Transport	13	10	
47	27.12.2002	Russia	Suicide bombing	Government	80	210	
48	19.05.2003	Israel	Suicide bombing	Facilities	3	52	
49	05.08.2003	Indonesia	Bombing	Facilities	10	150	
50	15.11.2003	Turkey	Bombing	Facilities	25	300	
51	15.11.2003	Colombia	Armed Assault	Personnel	1	72	
52	05.12.2003	Russia	Suicide bombing	Transport	42	150	
53	09.12.2003	Russia	Suicide bombing	Facilities	5	14	
54	29.01.2004	Israel	Suicide bombing	Transport	11	30	
55	01.02.2004	Iraq	Suicide bombing	Government	109	200	
56	22.02.2004	Israel	Suicide bombing	Transport	10	62	
57	11.03.2004	Spain	Bombing	Transport	191	1900	
58	09.05.2004	Russia	Bombing	Government	6	56	
59	04.08.2004	Greece	Bombing	Facilities	-	-	
60	22.09.2004	India	Armed Assault	Government		6	
61	22.09.2004	Israel	Suicide bombing	Facilities	2	16	
62	07.10.2004	Egypt	Bombing	Facilities	34	159	
63	24.10.2004	Turkey	Bombing	Facilities		6	
64	15.04.2005	Colombia	Armed Assault	Personnel	4	23	
65	18.04.2005	Russia	Armed Assault	Personnel	2	3	
66	10.05.2005	Russia	Bombing	Military	2	2	
67	10.05.2005	Colombia	Other	-	-	-	
68	23.06.2005	Australia	Other	-	-	-	
69	24.06.2005	Israel	Armed Assault	Facilities	5	12	
70	25.06.2005	Spain	Bombing	Facilities			
71	26.06.2005	Colombia	Armed Assault	Military	25		18
72	07.07.2005	UK	Suicide bombing	Transport	56	700	
73	23.07.2005	Egypt	Bombing	Facilities	88		
74	29.07.2005	Spain	Bombing	Facilities	-	-	
75	15.08.2005	Russia	Bombing	Personnel	1	7	
76	15.08.2005	Egypt	Bombing	Personnel		2	
77	25.08.2005	Russia	Bombing	Government	1	2	

Table 1 (Cont'd.): Terrorism Events

U.S.$ Billion	*Event*	*Victims*	*Year*	*Country*
32.5	9/11 Attacks	3,025	2001	U.S.A
20.9	Hurricane Andrew	43	1992	U.S.A
17.31	Northridge Earthquake	60	1994	U.S.A
7.6	Typhoon Mireille	51	1991	Japan
6.44	Winterstorm Daria	95	1990	France, UK et al
6.38	Winterstorm Lothar	110	1999	France, Switzerland et al
6.2	Hurricane Hugo	71	1989	Puerto Rico, U.S.A et al
4.84	Storms and floods	22	1987	France, UK et al
4.48	Winterstorm Vivian	64	1990	Western Europe
4.45	Typhoon Bart	26	1999	Japan

Source: OECD, 2005

Table 2. The 10 most costly insurance losses 1970-2003

Category	*Index*	*Number Of Observ.*
I. Global, European American and Swiss Stock Markets	FTSE All World	3054
	MSCI Europe	3054
	S&P 500	3051
	SMI	3054
II. Banks and Financials	FTSE Global Banks	2534
	FTSE Global Financials	2534
III. Insurance	MSCI Europe Insurance	2795
	FTSE All World Life Insurance	3054
	FTSE All World Non-Life Insurance	3054
IV. Travel and Airlines	FTSE All World Travel	3054
	MSCI Europe Airlines	2795
V. Defense and Pharmaceutical	FTSE All World Aero/Defense	3054
	FTSE All World Pharma/Biotech	3054
VI. Oil and Gas	FTSE All World Oil/Gas	3054
	FTSE Europe Oil/Gas	3054
VII. Commodity Markets	GSCI Commodity	3054
	GSCI Gold	3054
VIII. Bond Markets	J.P. Morgan Global Government Bond Index	3054
	FTSE Eurozone Bond Index	1925
	FTSE U.S. Government Bond Index	1491

Table 3. Financial Markets Indices. The table shows the list of stock indices considered in the study and the way they are grouped into eight categories.

	Min.	*Max.*	*Mean*	*S.D.*	*Skew.*	*Kurt.*	*Jarque-Bera*
FTSE All World	-0.0501	0.0466	0.0002	0.0083	-0.17	6.33	1427.59
MSCI Europe	-0.0635	0.0556	0.0002	0.0108	-0.26	6.47	1564.10
S&P 500	-0.0711	0.0557	0.0003	0.0107	-0.11	6.74	1776.40
SMI	-0.0733	0.0746	0.0003	0.0119	-0.15	7.38	2453.86
FTSE Global Banks	-0.0530	0.0707	0.0001	0.0116	0.03	6.14	1040.11
FTSE Global Financials	-0.0515	0.0684	0.0002	0.0115	0.13	5.92	903.20
MSCI Europe Insurance	-0.1287	0.0998	0.0002	0.0160	-0.33	9.58	5085.01
FTSE All World Life Insurance	-0.0618	0.0826	0.0004	0.0123	0.02	6.58	1630.26
FTSE All World Non-Life Insurance	-0.0610	0.0838	0.0003	0.0120	0.12	7.14	2185.27
FTSE All World Travel	-0.1291	0.1279	0.0001	0.0145	-0.23	10.03	6301.54
MSCI Europe Airlines	-0.1525	0.0899	0.0000	0.0157	-0.34	9.86	5711.73
FTSE All World Aero/Defense	-0.0858	0.0535	0.0003	0.0118	-0.47	7.38	2550.24
FTSE All World Pharma/Biotech	-0.0508	0.0595	0.0004	0.0097	-0.11	6.02	1162.56
FTSE All World Oil/Gas	-0.0686	0.0479	0.0004	0.0100	-0.32	6.03	1220.37
FTSE Europe Oil/Gas	-0.0776	0.0634	0.0004	0.0123	-0.16	5.83	1027.65
GSCI Commodity	-0.0915	0.0500	0.0003	0.0122	-0.20	5.25	662.83
GSCI Gold	-0.0508	0.0883	0.0000	0.0082	0.74	13.89	15359.69
J.P. Morgan GGBI	-0.0174	0.0189	0.0000	0.0039	0.06	4.54	301.77
FTSE Eurozone Bond Index	-0.0099	0.0074	0.0000	0.0021	-0.42	4.12	157.17
FTSE U.S. Gov. Bond Index	-0.0156	0.0138	0.0000	0.0031	-0.32	4.35	137.75

Table 4. Descriptive Statistics of Daily Returns on Different Stock Indices from January 1994 to August 2005. The table shows the descriptive statistics for daily returns on the stock indices. S.D. denotes standard deviation. *Skew.* denotes skewness, *Kurt.* denotes kurtosis. Jarque-Bera test (Jarque and Bera 1980) is a parametric hypothesis test of normality, equal to a sum of the standardized skewness and kurtosis. In our case Jarque-Bera test indicates that considered log transformed returns are not normally distributed. This is also equivalent to violation of the log-normality assumption of the original returns on index.

N	*Date*	*Natural Catastrophe*	*Country*	*Insured Loss (in U.S.D bn)*
1	04.01.1994	Northridge Earthquake	U.S.A	17.3
2	17.01.1995	Great Hanshin Earthquake	Japan	3
3	24-25.10.98	Winter Storm Winnie	UK, Netherlands Germany , Switzerland	0.2
4	17.08.1999	Earthquake in Izmit	Turkey	2
5	25.12.1999	Winter Storm Lothar	France, Switzerland et al	6.4
6	13-19.09.00	Typhoon Saomai	Japan, Korea, Russian	0.9
7	05.06.2001	Tropical Storm Alison	U.S.A	3.2
8	02.05.2003	Tornadoes and hail	U.S.A	3.2
9	26.12.2004	Tsunami	South Asia	12
10	01.08.2005	Hurricane Katrina	U.S.A	45

Table 5. Natural Catastrophes

N	Date of Attack	Country	Type of Attack	Target	Killed	Wounded	Global Market FTSE All World		European Market MSCI Europe		American Market S&P 500		Swiss Market SMI	
							ED	PEW	ED	PEW	ED	PEW	ED	PEW
1	11.09.2001	U.S.A	Bombing	Facilities	3000		***	**	***	**	***	***	***	**
2	15.11.2003	Turkey	Bombing	Facilities	25	300	**		***				**	
3	11.03.2004	Spain	Bombing	Transport	191	1900	***	*	***	***	***		***	***
4	09.05.2004	Russia	Bombing	Government	6	56	***	***	***	**	*		***	***
5	22.09.2004	Israel	Suicide Bombing	Facilities	2	16	***	***	*	**	**	**		*
6	07.10.2004	Egypt	Bombing	Facilities	34	159	*	***		**	**	**	**	***
7	24.10.2004	Turkey	Bombing	Facilities		6	*		***				**	
8	15.04.2005	Colombia	Armed Assault	Personnel	4	23	***		***	**	**			*
9	18.04.2005	Russia	Armed Assault	Personnel	2	3	**		***				***	*
10	07.07.2005	UK	Suicide Bombing	Transport	56	700	*		***				**	

Table 6. Database

	Terrorist Attacks		Financial Crashes		Natural Disasters	
Impact	Negative	Positive	Negative	Positive	Negative	Positive
FTSE All World	+		+		+	
MSCI Europe	+		+		+	
S&P500	+		+		+	
SMI	+		+		+	
FTSE Global Banks	+		+		+	
FTSE Global Financials	+		+		+	
MSCI Europe Insurance	+		+		+	
FTSE All World Life Insurance	+		+		+	
FTSE All World Non-Life Insurance	+		+		+	
FTSE All World Travel	+		+		+	
MSCI Europe Airlines	+		+		+	
FTSE All World Aero/Defense	+	+	+		+	+
FTSE All World Pharma/Biotech	+	+	+	+	+	+
FTSE All World Oil/Gas	+	+	+	+	+	+
FTSE Europe Oil/Gas	+	+	+	+	+	+
GSCI Commodity	+	+		+	+	+
GSCI Gold	+	+		+	+	+
J.P.Morgan GGBI	+	+		+	+	+
FTSE Eurozone Bond Index	+	+		+	+	+
FTSE U.S. Government Bond Index	+	+			+	

Table 7. Impact of Terrorist Attacks, Financial Crashes and Natural Disasters on Financial Markets

Index	Impact of Terrorist Events					Impact of Financial Crises					Impact of Natural Catastrophes				
	Event Day	Post-Event Window	Both	Total	Num. Events	Event Day	Post-Event Window	Both	Total	Num. Events	Event Day	Post-Event Window	Both	Total	Num. Events
FTSE All World	19	13	11	43	77	1	0	1	2	4	1	4	0	5	18
MSCI Europe	18	1	11	30	77	0	0	1	1	4	2	4	1	7	19
S&P500	11	4	8	23	77	2	0	0	2	4	0	1	0	1	19
SMI	15	13	12	40	77	0	0	1	1	4	1	6	0	7	19
FTSE Global Banks	12	5	8	25	67	1	0	1	2	3	0	1	1	2	17
FTSE Global Financials	14	9	9	32	67	1	0	1	2	3	0	4	0	4	17
MSCI Europe Insurance	16	8	13	37	75	0	0	1	1	3	1	1	1	3	17
FTSE All World Life Insurance	17	9	9	35	77	1	0	1	2	4	1	3	0	4	18
FTSE All World Non-Life Insurance	15	13	8	36	77	1	0	1	2	4	2	2	1	5	18
FTSE All World Travel	11	12	8	31	77	2	0	0	2	4	2	4	0	6	18
MSCI Europe Airlines	8	6	17	31	75	0	0	1	1	3	1	3	0	4	17
FTSE All World Aero/Defense	16	8	5	29	77	2	0	0	2	4	0	6	0	6	18
FTSE All World Pharma/Biotech	8	8	12	28	77	2	0	0	2	4	0	2	0	2	18
FTSE All World Oil/Gas	8	13	5	26	77	1	0	0	1	4	0	1	1	2	18
FTSE Europe Oil/Gas	9	10	1	20	77	1	0	0	1	4	1	3	1	5	18

Table 8. Event-Study Approach: Negative Effect of Terrorist Attacks, Financial Crashes and Natural Catastrophes on Stock Markets by Type of Impact. The table describes the effect of terrorist events, financial crashes and natural catastrophes on different stock markets. It shows the number of terrorist attacks, financial crashes and natural catastrophes that effected the performance of market indices either on the event-day or in the post-event window as well as in both periods.

Index	Terrorist Attacks				Financial Crashes				Natural Catastrophes			
	Event-Day Impact		*Post-Event* Window Impact		Event-Day Impact		*Post-Event* Window Impact		Event-Day Impact		*Post-Event* Window Impact	
	AR^{*}	$AR^{**/***}$	CAR^{*}	$CAR^{**/***}$	AR^{*}	$AR^{**/***}$	CAR^{*}	$CAR^{*/***}$	AR^{*}	$AR^{**/***}$	CAR^{*}	$CAR^{**/***}$
FTSE All World	15	15	11	13	0	2	0	1	1	0	1	3
MSCI Europe	7	22	2	11	0	1	0	1	0	2	2	3
S&P500	5	14	4	8	0	2	0	0	0	0	0	2
SMI	5	23	11	14	0	1	1	0	0	1	1	5
FTSE Global Banks	7	13	5	8	1	1	1	0	1	0	0	2
FTSE Global Financials	6	17	5	13	0	2	1	0	0	0	1	2
MSCI Europe Insurance	9	20	8	13	0	1	0	1	0	2	0	2
FTSE All World Life Insurance	7	20	7	13	0	2	1	0	1	0	1	2
FTSE All World Non-Life Insurance	11	13	4	17	0	2	0	1	2	1	0	3
FTSE All World Travel	7	12	8	12	0	2	0	0	1	1	2	2
MSCI Europe Airlines	5	20	8	15	0	1	1	0	0	1	1	2
FTSE All World Aero/Defense	5	16	4	9	0	2	0	0	0	0	3	3
FTSE All World Pharma/Biotech	5	15	8	12	0	2	0	0	0	0	0	2
FTSE All World Oil/Gas	3	10	9	9	0	1	0	0	1	0	1	1
FTSE Europe Oil/Gas	0	10	6	5	0	1	0	0	1	1	1	3

Table 9. Event-Study Approach: Negative Effect of Terrorist Attacks, Financial Crashes and Natural Catastrophes on Stock Markets by Strength of Impact. The table describes the effect of terrorist events, financial crashes and natural catastrophes on stock markets. It shows the strength of the impact of these extreme events. *AR* stands for the abnormal return and *CAR* for cumulative abnormal return. The number of stars next to AR and CAR indicates their statistical significance level: one star corresponds to 0.10, two stars to 0.05 and three stars to 0.01.

Index	Event Day			Post-Event Window			Both					Total
	AR^{*}	$AR^{**/***}$	Total	CAR^{*}	$CAR^{**/***}$	Total	AR^{*}	$AR^{**/***}$	CAR^{*}	$CAR^{**/***}$	Total	
Terrorist Attacks												
FTSE All World Aero/Defense	2 (2)	4 (5)	**6 (7)**	1 (0)	0 (0)	**1 (0)**	1 (0)	0 (0)	1 (0)	0 (0)	**1 (0)**	**8 (7)**
FTSE All World Pharma/Biotech	5 (1)	1 (5)	**6 (6)**	6 (2)	2 (3)	**8 (5)**	0 (1)	2 (2)	0 (1)	2 (2)	**2 (3)**	**16 (14)**
Financial Crashes												
FTSE All World Aero/Defense	0 (0)	0 (0)	**0 (0)**	0 (0)	0 (0)	**0 (0)**	0 (0)	0 (0)	0 (0)	0 (0)	**0 (0)**	**0 (0)**
FTSE All World Pharma/Biotech	0 (0)	0 (1)	**0 (1)**	1 (0)	0 (0)	**1 (0)**	0 (0)	0 (0)	0 (0)	0 (0)	**0 (0)**	**1 (1)**
Natural Catastrophes												
FTSE All World Aero/Defense	0 (0)	2 (1)	**2 (1)**	1 (0)	0 (1)	**1 (1)**	0 (0)	0 (1)	0 (1)	0 (0)	**0 (1)**	**3 (3)**
FTSE All World Pharma/Biotech	0 (0)	0 (0)	**0 (0)**	2 (1)	2 (2)	**4 (3)**	0 (0)	0 (1)	0 (0)	0 (1)	**0 (1)**	**4 (4)**

Table 10. Event-Study and Non-Parametric Approaches: Positive Impact of Terrorist Attacks, Financial Crashes and Natural Catastrophes on the Aero/Defense and Pharma/Biotech Sectors: by Type and Strength of Impact. The table describes the positive impact of different extreme events on aero/defense and pharma/biotech sectors. The values in parentheses correspond to the results of the non-parametric approach.

Index	Event Day			Post-Event Window			Both					Total
	AR^{*}	$AR^{**/***}$	Total	CAR^{*}	$CAR^{**/***}$	Total	AR^{*}	$AR^{**/***}$	CAR^{*}	$CAR^{**/***}$	Total	
Terrorist Attacks												
FTSE All World Oil/Gas	1 (6)	0 (1)	**1 (7)**	5 (2)	3 (1)	**8 (3)**	1 (1)	1 (0)	2 (1)	0 (0)	**2 (1)**	**11 (11)**
FTSE Europe Oil/Gas	2 (3)	0 (2)	**2 (5)**	2 (1)	2 (3)	**4 (4)**	1 (0)	1 (0)	1 (0)	1 (0)	**2 (0)**	**8 (9)**
Financial Crashes												
FTSE All World Oil/Gas	0 (0)	0 (0)	**0 (0)**	0 (1)	1 (0)	**1 (1)**	0 (0)	0 (0)	0 (0)	0 (0)	**0 (0)**	**1 (1)**
FTSE Europe Oil/Gas	0 (0)	0 (1)	**0 (1)**	0 (1)	1 (0)	**1 (1)**	0 (0)	0 (0)	0 (0)	0 (0)	**0 (0)**	**1 (2)**
Natural Catastrophes												
FTSE All World Oil/Gas	0 (0)	0 (0)	**0 (0)**	0 (1)	3 (1)	**3 (2)**	1 (1)	0 (1)	1 (1)	0 (1)	**1 (2)**	**4 (4)**
FTSE Europe Oil/Gas	0 (1)	0 (0)	**0 (1)**	2 (0)	1 (0)	**3 (0)**	0 (0)	0 (2)	0 (0)	0 (2)	**0 (2)**	**3 (3)**

Table 11. Event-Study and Non-Parametric Approaches: Positive Impact of Terrorist Attacks, Financial Crashes and Natural Catastrophes on the Oil and Gas Sector: by Type and Strength of Impact. The table describes the positive impact of different extreme events on the oil/gas sector. The values in parentheses correspond to the results of the non-parametric approach.

Index	Impact of Terrorist Events					Impact of Financial Crises					Impact of Natural Catastrophes				
	Event Day	Post-Event Window	Both	Total	Num. Events	Event Day	Post-Event Window	Both	Total	Num. Events	Event Day	Post-Event Window	Both	Total	Num. Events
GSCI Commodity	5	3	2	10	77	0	2	0	2	4	1	2	2	5	19
GSCI Gold	4	9	0	13	77	2	1	0	3	4	1	1	0	2	19
J.P. Morgan GGBI	9	7	4	20	77	0	0	2	2	4	1	0	2	3	19
FTSE Eurozone Bond Index	8	4	6	18	63	0	0	0	0	2	1	1	1	3	15
FTSE U.S. Gov. Bond Index	8	8	0	16	59	0	0	0	0	1	0	0	0	0	9

Table 12. Event-Study Approach: Positive Effect of Terrorist Attacks, Financial Crashes and Natural Catastrophes on Commodity and Bond Markets by Type of Impact. The table describes the effect of terrorist events, financial crashes and natural catastrophes on commodity and bond markets. It shows the number of terrorist attacks, financial crashes and natural catastrophes that effected the performance of market indices either on the event-day or in the post-event window as well as in both periods.

Index	Terrorist Attacks				Financial Crashes				Natural Catastrophes			
	Event-Day Impact		*Post-Event* Window Impact		Event-Day Impact		*Post-Event* Window Impact		Event-Day Impact		*Post-Event* Window Impact	
	AR^{*}	$AR^{**/***}$	CAR^{*}	$CAR^{**/***}$	AR^{*}	$AR^{**/***}$	CAR^{*}	$CAR^{*/***}$	AR^{*}	$AR^{**/***}$	CAR^{*}	$CAR^{**/***}$
GSCI Commodity	3	4	4	1	0	0	0	2	1	2	2	2
GSCI Gold	2	2	4	5	1	1	0	1	0	1	0	1
J.P. Morgan GGBI	4	9	6	5	0	2	0	2	0	3	1	1
FTSE Eurozone Bond Index	7	7	2	8	0	0	0	0	1	1	0	2
FTSE U.S. Government Bond Index	5	3	3	5	0	0	0	0	0	0	0	0

Table 13. Event-Study Approach: Positive Effect of Terrorist Attacks, Financial Crashes and Natural Catastrophes on Commodity and Bond Markets by Strength of Impact. The table describes the effect of terrorist events, financial crashes and natural catastrophes on commodity and bond markets. It shows the strength of the impact of these extreme events. AR stands for the abnormal return and CAR for the cumulative abnormal return. The number of stars next to AR and CAR indicates their statistical significance level: one star corresponds to 0.10, two stars to 0.05 and three stars to 0.01.

Index	Impact of Terrorist Events					Impact of Financial Crises					Impact of Natural Catastrophes				
	Event Day	Post-Event Window	Both	Total	Num. Events	Event Day	Post-Event Window	Both	Total	Num. Events	Event Day	Post-Event Window	Both	Total	Num. Events
GSCI Commodity	3	7	5	15	77	0	2	0	2	4	3	0	1	4	19
GSCI Gold	3	9	6	18	77	0	0	0	0	4	0	1	0	1	19
J.P. Morgan GGBI	2	2	2	6	77	0	0	0	0	4	2	0	1	3	19
FTSE Eurozone Bond Index	3	4	1	8	63	0	0	1	1	2	2	1	0	3	15
FTSE U.S. Gov. Bond Index	4	0	3	7	59	0	0	0	0	1	0	1	0	1	9

Table 14. Event-Study Approach: Negative Effect of Terrorist Attacks, Financial Crashes and Natural Catastrophes on Commodity and Bond Markets by Type of Impact. The table describes the effect of terrorist events, financial crashes and natural catastrophes on commodity and bond markets. It shows the number of terrorist attacks, financial crashes and natural catastrophes that effected the performance of market indices either on the event-day or in the post-event window as well as in both periods.

Index	Terrorist Attacks				Financial Crashes				Natural Catastrophes			
	Event-Day Impact		*Post-Event* Window Impact		Event-Day Impact		*Post-Event* Window Impact		Event-Day Impact		*Post-Event* Window Impact	
	AR^{*}	$AR^{**/***}$	CAR^{*}	$CAR^{**/***}$	AR^{*}	$AR^{**/***}$	CAR^{*}	$CAR^{*/***}$	AR^{*}	$AR^{**/***}$	CAR^{*}	$CAR^{**/***}$
GSCI Commodity	3	5	9	3	0	0	0	2	0	4	0	1
GSCI Gold	2	7	1	14	0	0	0	0	0	0	1	0
J.P. Morgan GGBI	1	3	1	3	0	0	0	0	0	3	0	1
FTSE Eurozone Bond Index	1	3	3	2	0	1	1	0	2	0	1	0
FTSE U.S. Government Bond Index	5	2	0	3	0	0	0	0	0	0	1	0

Table 15. Event-Study Approach: Negative Effect of Terrorist Attacks, Financial Crashes and Natural Catastrophes on Commodity and Bond Markets by Strength of Impact. The table describes the effect of terrorist events, financial crashes and natural catastrophes on commodity and bond markets. It shows the strength of the impact of these extreme events. AR stands for the abnormal return and CAR for the cumulative abnormal return. The number of stars next to AR and CAR indicates their statistical significance level: one star corresponds to 0.10, two stars to 0.05 and three stars to 0.01.

Index	Impact of Terrorist Events					Impact of Financial Crises					Impact of Natural Catastrophes				
	Event Day	Post-Event Window	Both	Total	Num. Events	Event Day	Post-Event Window	Both	Total	Num. Events	Event Day	Post-Event Window	Both	Total	Num. Events
FTSE All World	24	2	4	30	77	2	0	0	2	4	2	1	0	3	18
MSCI Europe	24	4	4	32	77	0	0	1	1	4	2	3	0	5	19
S&P500	18	6	5	29	77	2	0	0	2	4	0	1	0	1	19
SMI	20	4	9	33	77	1	0	0	1	4	1	2	0	3	19
FTSE Global Banks	20	1	1	22	66	1	0	0	1	3	0	0	0	0	16
FTSE Global Financials	20	6	3	29	66	1	0	0	1	3	1	0	0	1	16
MSCI Europe Insurance	25	5	6	36	73	1	0	0	1	3	2	1	0	3	17
FTSE All World Life Insurance	23	2	4	29	76	1	0	0	1	4	1	2	0	3	18
FTSE All World Non-Life Insurance	19	6	5	30	76	1	0	0	1	4	3	1	0	4	18
FTSE All World Travel	20	4	0	24	76	2	0	0	2	4	0	0	0	0	18
MSCI Europe Airlines	12	3	9	24	73	1	0	0	1	3	0	0	0	0	17
FTSE All World Aero/Defense	20	6	1	27	76	2	0	0	2	4	1	3	0	4	18
FTSE All World Pharma/Biotech	14	2	4	20	76	2	0	0	2	4	0	0	0	0	18
FTSE All World Oil/Gas	13	11	3	27	76	1	0	0	1	4	0	2	0	2	18
FTSE Europe Oil/Gas	11	4	2	17	76	1	0	0	1	4	1	2	0	3	18

Table 16. Non-Parametric Approach: Negative Effect of Terrorist Attacks, Financial Crashes and Natural Catastrophes on Stock Markets by Type of Impact. The table describes the effect of terrorist events, financial crashes and natural catastrophes on stock markets markets. It shows the number of terrorist attacks, financial crashes and natural catastrophes that effected the performance of market indices either on the event-day or in the post-event window as well as in both periods.

Index	Terrorist Attacks				Financial Crashes				Natural Catastrophes			
	Event-Day Impact		*Post-Event* Window Impact		*Event-Day* Impact		*Post-Event* Window Impact		*Event-Day* Impact		*Post-Event* Window Impact	
	Abn.	Extr.	Abn.	Extr.	Abn.	Extr.	Abn.	Extr.	Abn.	Extr.	Abn.	Extr.
FTSE All World	9	19	2	4	0	2	0	0	1	1	0	1
MSCI Europe	11	17	4	4	0	1	0	1	0	2	2	1
S&P500	4	19	6	5	0	2	0	0	0	0	1	0
SMI	7	22	5	8	0	1	0	0	0	1	0	2
FTSE Global Banks	10	11	1	1	0	1	0	0	0	0	0	0
FTSE Global Financials	5	19	6	4	0	1	0	0	0	1	0	0
MSCI Europe Insurance	13	18	5	6	0	1	0	0	0	2	0	1
FTSE All World Life Insurance	10	17	2	5	0	1	0	0	1	0	1	1
FTSE All World Non-Life Insurance	6	19	6	5	0	1	0	0	3	0	0	1
FTSE All World Travel	5	15	2	2	0	2	0	0	0	0	0	0
MSCI Europe Airlines	2	19	2	10	0	1	0	0	0	2	0	2
FTSE All World Aero/Defense	7	15	2	5	0	2	0	0	0	1	2	1
FTSE All World Pharma/Biotech	4	15	2	4	0	2	0	0	0	0	0	0
FTSE All World Oil/Gas	4	12	4	10	0	1	0	0	0	0	0	2
FTSE Europe Oil/Gas	5	8	5	1	1	0	0	0	0	1	0	2

Table 17. Non-Parametric Approach: Negative Effect of Terrorist Attacks, Financial Crashes and Natural Catastrophes on Stock Markets by Strength of Impact. The table describes the effect of terrorist events, financial crashes and natural catastrophes on stock markets. It shows the strength of the impact of these extreme events. *Abn.* stands for abnormal movements in the event-day returns or in the post-event window (conditional probability is in the interval (0.05;0.10]). *Extr.* corresponds to extreme index movements (conditional probability is in the interval [0.00;0.05]).

Index	Impact of Terrorist Events					Impact of Financial Crises					Impact of Natural Catastrophes				
	Event Day	Post-Event Window	Both	Total	Num. Events	Event Day	Post-Event Window	Both	Total	Num. Events	Event Day	Post-Event Window	Both	Total	Num. Events
GSCI Commodity	8	6	0	14	77	0	1	0	1	4	2	3	1	6	19
GSCI Gold	3	6	3	12	77	0	1	1	2	4	2	1	0	3	19
J.P. Morgan GGBI	3	4	7	14	77	0	0	2	2	4	2	2	0	4	19
FTSE Eurozone Bond Index	13	3	0	16	62	0	1	0	1	2	2	2	0	4	14
FTSE U.S. Gov. Bond Index	13	0	0	13	59	0	0	0	0	1	0	0	0	0	8

Table 18. Non-Parametric Approach: Positive Effect of Terrorist Attacks, Financial Crashes and Natural Catastrophes on Commodity and Bond Markets by Type of Impact. The table describes the effect of terrorist events, financial crashes and natural catastrophes on commodity and bond markets. It shows the number of terrorist attacks, financial crashes and natural catastrophes that effected the performance of market indices either on the event-day or in the post-event window as well as in both periods.

Index	Terrorist Attacks				Financial Crashes				Natural Catastrophes			
	Event-Day Impact		*Post-Event* Window Impact		*Event-Day* Impact		*Post-Event* Window Impact		*Event-Day* Impact		*Post-Event* Window Impact	
	Abn.	Extr.	Abn.	Extr.	Abn.	Extr.	Abn.	Extr.	Abn.	Extr.	Abn.	Extr.
GSCI Commodity	1	7	4	2	0	0	0	1	1	2	2	2
GSCI Gold	3	3	5	4	0	1	0	2	0	2	1	0
J.P. Morgan GGBI	2	8	3	8	0	2	0	2	1	1	0	2
FTSE Eurozone Bond Index	6	7	3	0	0	0	0	1	1	1	1	1
FTSE U.S. Government Bond Index	8	5	0	0	0	0	0	0	0	0	0	0

Table 19. Non-Parametric Approach: Positive Effect of Terrorist Attacks, Financial Crashes and Natural Catastrophes on Commodity and Bond Markets by Strength of Impact. The table describes the effect of terrorist events, financial crashes and natural catastrophes on commodity and bond markets. It shows the strength of the impact of these extreme events. *Abn.* stands for abnormal movements in the event-day returns or in the post-event window (conditional probability is in the interval (0.05;0.10]). *Extr.* corresponds to extreme index movements (conditional probability is in the interval [0.00;0.05]).

Index	Impact of Terrorist Events					Impact of Financial Crises					Impact of Natural Catastrophes				
	Event Day	Post-Event Window	Both	Total	Num. Events	Event Day	Post-Event Window	Both	Total	Num. Events	Event Day	Post-Event Window	Both	Total	Num. Events
GSCI Commodity	7	1	3	11	77	0	0	0	0	4	2	0	1	3	19
GSCI Gold	6	8	2	16	77	0	0	0	0	4	1	2	0	3	19
J.P. Morgan GGBI	3	5	1	9	77	0	0	0	0	4	1	0	1	2	19
FTSE Eurozone Bond Index	4	0	0	4	62	0	0	0	0	2	2	0	0	2	14
FTSE U.S. Gov. Bond Index	6	0	0	6	59	0	0	0	0	1	1	0	0	1	8

Table 20. Non-Parametric Approach: Negative Effect of Terrorist Attacks, Financial Crashes and Natural Catastrophes on Commodity and Bond Markets by Type of Impact. The table describes the effect of terrorist events, financial crashes and natural catastrophes on commodity and bond markets. It shows the number of terrorist attacks, financial crashes and natural catastrophes that effected the performance of market indices either on the event-day or in the post-event window as well as in both periods.

Index	Terrorist Attacks				Financial Crashes				Natural Catastrophes			
	Event-Day Impact		*Post-Event* Window Impact		*Event-Day* Impact		*Post-Event* Window Impact		*Event-Day* Impact		*Post-Event* Window Impact	
	Abn.	Extr.	Abn.	Extr.	Abn.	Extr.	Abn.	Extr.	Abn.	Extr.	Abn.	Extr.
GSCI Commodity	3	7	1	3	0	0	0	0	0	2	1	0
GSCI Gold	2	6	4	6	0	0	0	0	1	0	0	2
J.P. Morgan GGBI	2	2	4	2	0	0	0	0	1	1	0	2
FTSE Eurozone Bond Index	1	3	0	0	0	0	0	0	1	1	0	0
FTSE U.S. Government Bond Index	5	1	0	0	0	0	0	0	1	0	0	0

Table 21. Non-Parametric Approach: Negative Effect of Terrorist Attacks, Financial Crashes and Natural Catastrophes on Commodity and Bond Markets by Strength of Impact. The table describes the effect of terrorist events, financial crashes and natural catastrophes on commodity and bond markets. It shows the strength of the impact of these extreme events. *Abn.* stands for the abnormal movements in the event-day returns or in the post-event window (conditional probability is in the interval (0.05;0.10]). *Extr.* corresponds to extreme index movements (conditional probability is in the interval [0.00;0.05]).

	Event-Day Impact								
Index	Terrorist Attacks			Financial Crashes			Natural Catastrophes		
	Total	Extreme	Number of Events	Total	Extreme	Number of Events	Total	Extreme	Number of Events
FTSE All World	26	19	76	1	1	4	0	0	18
MSCI Europe	22	16	77	1	1	4	2	1	19
S&P500	22	15	77	2	2	4	0	0	19
SMI	28	15	77	1	1	4	1	1	19
FTSE Global Banks	20	10	69	0	0	2	0	0	17
FTSE Global Financials	21	19	66	2	1	3	0	0	17
MSCI Europe Insurance	28	20	73	1	1	3	2	0	17
FTSE All World Life Insurance	23	11	74	2	2	4	0	0	17
FTSE All World Non-Life Insurance	22	15	75	1	1	3	2	0	16
FTSE All World Travel	19	8	77	2	2	4	0	0	18
MSCI Europe Airlines	23	21	74	1	1	3	2	1	17
FTSE All World Aero/Defense	23	16	76	2	2	4	1	0	18
FTSE All World Pharma/Biotech	15	10	76	2	2	4	0	0	18
FTSE All World Oil/Gas	13	12	62	0	0	2	0	0	15
FTSE Europe Oil/Gas	10	8	62	0	0	2	1	0	15
GSCI Commodity	7	2	77	0	0	4	2	2	19
GSCI Gold	9	7	77	0	0	4	0	0	19
J.P.Morgan GGBI	4	3	77	0	0	4	2	2	19
FTSE Eurozone Bond Index	3	2	60	0	0	1	1	0	14
FTSE U.S. Government Bond Index	3	1	59	0	0	1	0	0	9

Table 22. GARCH-EVT Approach: Negative Effect of Terrorist Attacks, Financial Crashes and Natural Catastrophes on Financial Markets. The table describes the effect of terrorist events, financial crashes and natural catastrophes on different financial markets. It shows the number of terrorist attacks, financial crashes and natural catastrophes that effected the performance of market indices on the event-day.

	Event-Day Impact								
Index	Terrorist Attacks			Financial Crashes			Natural Catastrophes		
	Total	Extreme	Number of Events	Total	Extreme	Number of Events	Total	Extreme	Number of Events
FTSE All World Aero/Defense	6	6	76	0	0	4	1	1	18
FTSE All World Pharma/Biotech	9	6	76	1	1	4	0	0	18
FTSE All World Oil/Gas	4	1	62	0	0	2	1	0	15
FTSE Europe Oil/Gas	6	3	62	0	0	2	2	0	15
GSCI Commodity	10	5	77	0	0	4	2	2	19
GSCI Gold	2	2	77	2	1	4	0	0	19
J.P.Morgan GGBI	12	5	77	2	2	4	1	0	19
FTSE Eurozone Bond Index	12	8	60	0	0	1	2	2	14
FTSE U.S. Government Bond Index	7	4	59	0	0	1	0	0	9

Table 23. GARCH-EVT Approach: Positive Effect of Terrorist Attacks, Financial Crashes and Natural Catastrophes on Financial Markets. The table describes the positive effect of terrorist events, financial crashes and natural catastrophes on different financial markets. It shows the number of terrorist attacks, financial crashes and natural catastrophes that effected the performance of market indices on the event-day.

Index	Terrorist Attacks Event-Study			Terrorist Attacks Non-Parametric			Terrorist Attacks GARCH-EVT			Financial Crashes Event-Study			Financial Crashes Non-Parametric			Financial Crashes GARCH-EVT		
	Total	Ext.	Num. Events	Total	Ext.	Num. Events	Total	Ext.	Num. Events	Total	Ext.	Num. Events	Total	Ext.	Num. Events	Total	Ext.	Num. Events
FTSE All World	30	15	77	28	19	77	26	19	76	2	2	4	2	2	4	1	1	4
MSCI Europe	29	22	77	28	17	77	22	16	77	1	1	4	1	1	4	1	1	4
S&P500	19	14	77	23	19	77	22	15	77	2	2	4	2	2	4	2	2	4
SMI	27	23	77	29	22	77	28	15	77	1	1	4	1	1	4	1	1	4
FTSE Global Banks	20	13	67	21	11	66	20	10	69	2	1	3	1	1	3	0	0	2
FTSE Global Financials	23	17	67	23	19	66	21	19	66	2	2	3	1	1	3	2	1	3
MSCI Europe Insurance	29	20	75	31	18	73	28	20	73	1	1	3	1	1	3	1	1	3
FTSE All World Life Ins.	26	20	77	27	17	76	23	11	74	2	2	4	1	1	4	2	2	4
FTSE All World Non-Life Insurance	23	13	77	24	19	76	22	15	75	2	2	4	1	1	4	1	1	3
FTSE All World Travel	19	12	77	20	15	76	19	8	77	2	2	4	2	2	4	2	2	4
MSCI Europe Airlines	25	20	75	21	19	73	23	21	74	1	1	3	1	1	3	1	1	3
FTSE All World Aero/Def.	21	16	77	21	15	76	23	16	76	2	2	4	2	2	4	2	2	4
FTSE All World Pharma/Bio	20	15	77	18	15	76	15	10	76	2	2	4	2	2	4	2	2	4
FTSE All World Oil/Gas	13	10	77	16	12	76	13	12	62	1	1	4	1	1	4	0	0	2
FTSE Europe Oil/Gas	10	10	77	13	8	76	10	8	62	1	1	4	1	0	4	0	0	2
GSCI Commodity	8	5	77	10	7	77	7	2	77	0	0	4	0	0	4	0	0	4
GSCI Gold	9	7	77	8	6	77	9	7	77	0	0	4	0	0	4	0	0	4
J.P.Morgan GGBI	4	3	77	4	4	77	4	3	77	0	0	4	0	0	4	0	0	4
FTSE Eurozone Bond Index	4	3	63	4	3	62	3	2	60	1	1	2	0	0	2	0	0	1
FTSE U.S. Gov. Bond Index	7	2	59	6	1	59	3	1	59	0	0	1	0	0	1	0	0	1

Table 24. Comparison of the Event-Day Negative Impact of Terrorist Attacks, Financial Crashes and Natural Catastrophes across Different Methodologies.

Index				Natural Catastrophes					
	Event-Study			Non-Parametric			GARCH-EVT		
	Total	Ext.	Num. Events	Total	Ext.	Num. Events	Total	Ext.	Num. Events
FTSE All World	1	0	18	2	1	18	0	0	18
MSCI Europe	3	2	19	2	2	19	2	1	19
S&P500	0	0	19	0	0	19	0	0	19
SMI	1	1	19	1	1	19	1	1	19
FTSE Global Banks	1	0	17	0	0	16	0	0	17
FTSE Global Financials	0	0	17	1	1	16	0	0	17
MSCI Europe Insurance	2	2	17	2	2	17	2	0	17
FTSE All World Life Insurance	1	0	18	1	0	18	0	0	17
FTSE All World Non-Life	3	1	18	3	0	18	2	0	16
FTSE All World Travel	2	1	18	0	0	18	0	0	18
MSCI Europe Airlines	1	1	17	0	2	17	2	1	17
FTSE All World Aero/Defense	0	0	18	1	1	18	1	0	18
FTSE All World Pharma/Biotech	0	0	18	0	0	18	0	0	18
FTSE All World Oil/Gas	1	0	18	0	0	18	0	0	15
FTSE Europe Oil/Gas	2	1	18	1	1	18	1	0	15
GSCI Commodity	4	4	19	3	2	19	2	2	19
GSCI Gold	0	0	19	1	0	19	0	0	19
J.P.Morgan GGBI	3	3	19	2	1	19	2	2	19
FTSE Eurozone Bond Index	2	0	15	2	1	14	1	0	14
FTSE U.S. Government Bond Index	0	0	9	1	0	8	0	0	9

Table 24 (Cont'd.). Comparison of the Event-Day Negative Impact of Terrorist Attacks, Financial Crashes and Natural Catastrophes across Different Methodologies.

Index	Terrorist Attacks Event-Study			Terrorist Attacks Non-Parametric			Terrorist Attacks GARCH-EVT			Financial Crashes Event-Study			Financial Crashes Non-Parametric			Financial Crashes GARCH-EVT		
	Total	Ext.	Num. Events	Total	Ext.	Num. Events	Total	Ext.	Num. Events	Total	Ext.	Num. Events	Total	Ext.	Num. Events	Total	Ext.	Num. Events
FTSE All World Aero/Def.	6	4	77	7	5	76	6	6	76	0	0	4	0	0	4	0	0	4
FTSE All World Pharma/ Biotech	6	1	77	6	5	76	9	6	76	0	0	4	1	1	4	1	1	4
FTSE All World Oil/Gas	1	0	77	7	1	76	4	1	62	0	0	4	0	0	4	0	0	2
FTSE Europe Oil/Gas	2	0	77	5	2	76	6	3	62	0	0	4	1	1	4	0	0	2
GSCI Commodity	7	4	77	8	7	77	10	5	77	0	0	4	0	0	4	0	0	4
GSCI Gold	4	2	77	6	3	77	2	2	77	2	1	4	1	1	4	2	1	4
J.P.Morgan GGBI	13	9	77	10	8	77	12	5	77	2	2	4	2	2	4	2	2	4
FTSE Eurozone BI	14	7	63	13	7	62	12	8	60	0	0	2	0	0	2	0	0	1
FTSE U.S. Gov. Bond Index	8	3	59	13	5	59	7	4	59	0	0	1	0	0	1	0	0	1

Table 25. Comparison of the Event-Day Positive Impact of Terrorist Attacks, Financial Crashes and Natural Catastrophes across Different Methodologies.

Index	Natural Catastrophes Event-Study			Natural Catastrophes Non-Parametric			Natural Catastrophes GARCH-EVT		
	Total	Ext.	Num. Events	Total	Ext.	Num. Events	Total	Ext.	Num. Events
FTSE All World Aero/Defense	2	2	18	1	1	18	1	1	18
FTSE All World Pharma/Biotech	0	0	18	0	0	18	0	0	18
FTSE All World Oil/Gas	0	0	18	0	0	18	1	0	15
FTSE Europe Oil/Gas	0	0	18	1	0	18	2	0	15
GSCI Commodity	3	2	19	3	2	19	2	2	19
GSCI Gold	1	1	19	2	2	19	0	0	19
J.P.Morgan GGBI	3	3	19	2	1	19	1	0	19
FTSE Eurozone Bond Index	2	1	15	2	1	14	2	2	14
FTSE U.S. Government Bond Index	0	0	9	0	0	8	0	0	9

Table 25 (Cont'd.) Comparison of the Event-Day Positive Impact of Terrorist Attacks, Financial Crashes and Natural Catastrophes across Different Methodologies.

			FTSE All World Aero/Defense			FTSE All World Pharma/Biotech		
N	Date	Attack	Event Study	Non-Parametric A.	GARCH-EVT	Event Study	Non-Parametric A.	GARCH-EVT
1	19.04.1995	Bombing in Oklahoma City	+	+	+			
2	08.01.1996	Kidnapping in Indonesia	+	+	+			
3	31.01.1996	Bomb Attack in Sri Lanka				+	+	
4	04.03.1996	Suicide Bombing in Israel	+	+	+	+	+	+
5	10.04.2002	Armed Assault in India	+	+	+		+	+
6	10.04.2002	Suicide Bombing in Israel	+	+	+		+	+
7	08.05.2002	Bombing in Pakistan	+	+	+		+	+
8	17.07.2002	Suicide Bombing in Israel	+	+				
9	04.08.2002	Bombing in Israel				+	+	
10	05.08.2002	Armed Assault in Pakistan				+	+	
11	12.10.2002	Bombing in Indonesia/Bali				+	+	+
12	27.12.2002	Suicide Bombing in Russia				+	+	
13	01.02.2004	Suicide Bombing in Iraq				+	+	+
14	15.04.2005	Armed Assault in Colombia				+	+	+

Table 26. Positive effect on the Aero/Defense and Pharma/Biotech Industries: Common Terrorist Events.

			FTSE Europe Oil/Gas			FTSE World Oil/Gas		
N	Date	Attack	Event Study	Non-Parametric A.	GARCH-EVT	Event Study	Non-Parametric A.	GARCH-EVT
1	17.04.1999	Bombing in the UK		+	+		+	
2	11.09.2001	9/11 Attacks in the U.S.	+	+	+	+		
3	08.05.2002	Bombing in Pakistan	+			+	+	+
4	17.07.2002	Suicide Bombing in Israel	+	+		+	+	
5	31.07.2002	Bombing in Israel		+		+	+	
6	04.08.2002	Bombing in Israel		+		+	+	
7	05.08.2002	Armed Assault in Pakistan		+		+	+	
8	05.08.2003	Bombing in Indonesia/Bali	+	+		+	+	
9	05.12.2003	Suicide Bombing in Russia	+			+		
10	09.12.2003	Suicide Bombing in Russia	+	+	+	+	+	+
11	18.04.2005	Armed Assault in Russia				+	+	
12	25.08.2005	Bombing in Russia		+		+	+	

Table 27. Positive Effect on the Oil/Gas Industry: Common Terrorist Events.

Index	Event Day			Post-Event Window			Both					Total
Terrorist Attacks												
	AR*	AR**/***	Total	CAR*	CAR**/***	Total	AR*	AR**/***	CAR*	CAR**/***	Total	
DJ Kuwait Titans 50	1 (2)	3 (2)	**4 (4)**	3 (5)	5 (3)	**8 (8)**	0 (2)	6 (2)	3 (0)	3 (4)	**6 (4)**	**18 (16)**
Financial Crashes												
	AR*	AR**/***	Total	CAR*	CAR**/***	Total	AR*	AR**/***	CAR*	CAR**/***	Total	
DJ Kuwait Titans 50	0 (0)	1 (1)	**1 (1)**	0 (0)	1(0)	**1 (0)**	0 (0)	0 (0)	0 (0)	0 (0)	**0 (0)**	**2 (1)**
Natural Catastrophes												
	AR*	AR**/***	Total	CAR*	CAR**/***	Total	AR*	AR**/***	CAR*	CAR**/***	Total	
DJ Kuwait Titans 50	0 (1)	3 (4)	**3 (5)**	0 (1)	0 (0)	**0 (1)**	0 (0)	2 (0)	1 (0)	1 (0)	**2 (0)**	**5 (6)**

Table 28. Event-Study and Non-Parametric Approaches: Negative Impact of Terrorist Attacks, Financial Crashes and Natural Catastrophes on the Dow Jones Kuwait Titans 50: by Type and Strength of Impact. The table describes a negative impact of different extreme events on the Dow Jones Kuwait Titans 50 stock index. The values in parentheses correspond to the results of the non-parametric approach.

Index	Event Day			Post-Event Window			Both					Total
Terrorist Attacks												
	AR^{*}	$AR^{**/***}$	Total	CAR^{*}	$CAR^{**/***}$	Total	AR^{*}	$AR^{**/***}$	CAR^{*}	$CAR^{**/***}$	Total	
DJ Kuwait Titans 50	2 (2)	4 (5)	**6 (7)**	1 (0)	0 (0)	**1 (0)**	1 (0)	0 (0)	1 (0)	0 (0)	**1 (0)**	**8 (0)**
Natural Catastrophes												
	AR^{*}	$AR^{**/***}$	Total	CAR^{*}	$CAR^{**/***}$	Total	AR^{*}	$AR^{**/***}$	CAR^{*}	$CAR^{**/***}$	Total	
DJ Kuwait Titans 50	0 (1)	0 (1)	**0 (2)**	0 (0)	0 (0)	**0 (0)**	0 (0)	1 (0)	0 (0)	1 (0)	**1 (0)**	**1 (2)**

Table 29. Event-Study and Non-Parametric Approaches: Positive Impact of Terrorist Attacks, Financial Crashes and Natural Catastrophes on the Dow Jones Kuwait Titans 50: by Type and Strength of Impact. The table describes a positive impact of different extreme events on the Dow Jones Kuwait Titans 50 stock index. The values in parentheses correspond to the results of the non-parametric approach.

Paper 3

Pricing of Multiple-Event Coupon Paying CAT Bond

3.1 Introduction

The recent history of natural disasters and terrorist events reveals a changing nature of these catastrophic events both in terms of their frequency and their magnitude[1]. Globally, weather-related disasters occur at over five times the rate that they did 40 years ago. Economic losses due to natural disasters appear to be doubling every 10 years, and by the next decade they have been forecast to reach \$150 billion per year (Fishel (2005)). As to terrorism risk, the long-term outlook on this risk remains negative and insurers with big natural catastrophe cover are looking to terrorism as a new area to underwrite[2]. At year-end 2005, nearly two-thirds of businesses in the US had purchased terrorism risk insurance policies. Real estate firms, financial institutions, health care facilities and media companies were buyers of this type of coverage (Valverde and Hartwig (2006)). The anticipated increase of severe storms and weather events associated with climate

[1]For example, the hurricane Katrina in August 2005 was one of the deadliest natural disasters in U.S. history with estimated 1,353 direct fatalities, \$40.6 billion in insured losses and more than \$100 billion in economic losses (Johnson (2006)). As to terrorist events, the 9/11 attacks led to $2,976$ deaths and inflicted damage currently estimated at around USD 200 billion with \$35.6 billion attributed to insurance claim payments (Valverde and Hartwig (2006)). Another example is the bombings in Madrid in 2004 that resulted in 191 people killed and \$110.9 million insurance cost (SwissRe (2006)).

[2]According to Lloyds, London market insurers have been told that the long-term outlook on terrorism is negative, with attacks likely to diversify away from the transport network to places of public entertainment and other major cities such as Manchester and Birmingham. Besides the UK, Italy, the Netherlands, Spain and France are most at risk of an attack in Western Europe. Those at risk, but to a lesser degree, include Germany, Belgium, Denmark and Norway (Lloyd's (2006)).

change and on-going threat of terrorist events can place enormous financial demands on the insurance and reinsurance businesses (Fishel (2005)). As a result, development of the ways to transfer these catastrophic risks to capital markets has become more important than ever before. While issuance of risk-linked securities (ILSs) such as catastrophe bonds (CAT bonds) has played an important role in managing the exposure to the risk from natural catastrophes, securitization of terrorism risk in a similar way has become a real challenge. The latter is due to inherent uncertainties associated with terrorism risk and consequent difficulties in modelling and quantification of this risk. The feedback from reinsurance industry shows that, if securitized, terrorism risk is still preferred to be packaged together with other catastrophic risks. From the point of view of investors, rating agencies and issuing companies themselves, ILSs constructed this way seem to have a better potential to find its niche on the market.

This study is the first of its kind that focuses on the issue of pricing of a multiple-event coupon paying CAT bond that covers exposure to catastrophic risks, including terrorism. Securitization of the latter by means of CAT bonds with a multiple-event and multiple-risk structure is suggested in some of the literature (see OECD (2005) and Woo (2004)). However, none of the existing research develops a theoretical model to price such instruments. This study is the first to address this issue. We implement theoretical pricing of a multiple-event CAT bond in incomplete market setting using a representative agent pricing model. In contrast to the existing literature on pricing of standard CAT bonds, the payoffs on the bond in our model are linked to two types of underlying processes: catastrophic property losses and catastrophic mortality. Along with natural catastrophes and man-made disasters, our model views terrorism risk as one of the main exposure risks that may effect the cash flows of the bond. To capture the impact of terrorist attacks we focus on daily instead of annual catastrophe mortality. We set attachment levels that allow us to capture possible levels of terrorism risk causalities. Finally, we use a multiple-event contingency of the bond's cash flows that is a preferred choice with respect to terrorism risk securitization according to the literature (see OECD (2005)). We believe that this way of structuring the bond allows us to incorporate the impact of terrorist events in a better way compared to existing risk transfer transactions. For example, the Vita Capital catastrophe-indexed notes issued by Swiss Re in 2003 were aimed at hedging catastrophe mortality risk. The payoffs on the notes were linked to the value of the annual combined mortality index (the weighted average of annual population death rates in the US, UK, France, Italy and Switzerland) with such events under coverage as natural disasters, epidemics, terrorism and wars. The principal of these notes was at risk if, during any single calendar year in the risk coverage period, the mortality index exceeds 130% of its baseline 2002 level (equivalent to approximately more than 800,000 deaths according to the Standard and Poor's report (SP (2003))). In this setting, the impact of terrorism risk was diluted. Even the 9/11 attacks that resulted in almost 3000 deaths, on their own, would not be an event significant enough to trigger the cash flow risk of these notes.

An important contribution of this work is that it provides a numerical evaluation of the price of the bond under consideration. We implement a Monte Carlo simulation of the price using the UK catastrophe data provided by Swiss Re. We assume that both, the bond's coupons and its principal are at risk if the triggering events occur. We consider different cases/scenarious for which we compute corresponding prices of coupon-bearing and zero-coupon CAT bonds. The scenarios considered here include a possibility of events of the magnitude of the 9/11 attacks and higher as it can be in the case of chemical, biological, radiological and nuclear terrorist incidents.

The results of this study indicate that the price of the bond increases with threshold levels and decreases with stronger positive dependence between losses and deaths. As to time to maturity, there is an inverse relationship between the price of the bond and its time to expiration. Although this relationship always works for a zero-coupon catastrophe bond, it may not always hold when the bond pays coupons. In general, we find a less responsive behavior of the bond's price to changes in dependence between losses and deaths than to changes in the bond's time to maturity. In addition, we see that for a coupon paying CAT bond the impact of increase in attachment levels on the bond's price dominates the effect of extended maturity. The opposite result is observed for a zero-coupon CAT bond.

3.2 Related Research

When implementing our model we benefit from existing literature that can be grouped into several strands of research. The first strand of research describes pricing of standard CAT bonds. This group of literature is the most closely related to our paper. For example, Louberge, Kellezi, and Gilli (1999) numerically estimate the CAT bond price assuming Poisson and lognormal distributions for frequency and severity of catastrophe losses correspondingly, and binomial random process for interest rates. The paper by Lee and Yu (2002) describes a pricing model of default-risky CAT bond, accounting for moral hazard and basis risk[3]. The authors adopt the approach of Duan, Moreau, and Sealey (1995) to describe insurer's total asset value as consisting of two risk components: interest rate and credit risks. They assume the square-root process of Cox for the dynamics of interest rate and a compound Poisson process for the aggregate loss dynamics.

When pricing catastrophic risks, the market incompleteness is an important issue to be addressed. Cox and Pedersen (2000) propose a framework of pricing CAT bonds in the incomplete market setting. They capture this issue using a representative agent technique. Another possible approach to address incompleteness is by means of the so called Wang transform (Wang (2004)). This approach is used in the study by Lin and Cox (2006). The authors consider securitization of catastrophic mortality risk and use the Vita Capital transaction of Swiss Re as an example when developing the framework of their model. When modelling stochastic mortality behavior, they assume the Brownian motion dynamics for the unanticipated 'normal' mortality index change and a jump process for 'abnormal' mortality shocks.

Papers by Dahl (2004), Cairns, Blake, and Dowd (2004), Lin and Cox (2005) focus on non-catastrophe mortality risk and describe different pricing frameworks for securitization of this risk. Burnecki (2005) considers pricing of CAT bonds using a compound non-homogeneous Poisson model with left truncated loss distribution. The author considers

[3]The basis risk is the risk of the gap between the insurer's actual loss and the composite index of losses that prevent the insurer from receiving complete risk hedging. The basis risk may cause the default of insurer in case of high individual loss and therefore this risk affects the bond's price. The moral hazard problem arises when the insurer's cost of loss control efforts exceeds the benefits of debt forgiveness. Consequently, it may increase the claim payments at the expense of the bondholders coupon and principal reduction and affect the bond price. There is a tradeoff between these two problems. For example, if the parametric index is used to define the CAT bond payments then the moral hazard problem is reduced while the basis risk is created (Lee and Yu (2002)).

losses resulting from natural catastrophic events in the U.S.

Finally, Barrieu and Louberge (2007) analyze the effect of hybrid CAT bonds on the volume of capital flowing into the CAT bond market. Hybrid CAT bonds combine the transfer of catastrophic risk with protection against a stock market crash. The paper shows that replacing standard CAT bonds with hybrid CAT bonds would lead to increase in market volume, in particular when investors are strongly risk averse, compared to issuers of CAT bonds (insurers, reinsurers and large corporations).

Only few articles focus on pricing of catastrophe-linked securities other than CAT bonds. For example, Cummins and Geman (1995) and Chang, J.Chang, and M.Yu (1996) consider pricing of CAT futures and CAT call spreads. Another paper, by Aase (2001) presents a valuation framework, a Markov model, of futures contracts and derivatives on such contracts. Litzenberger, Beaglehole, and Reynolds (1996) explore approaches to the valuation, pricing and assessment of CAT options.

Alternative risk transfer instruments such as catastrophe-linked securities both exchange-traded and OTC are usually considered a good supplement to a more traditional approach of dealing with catastrophic risk, namely reinsurance. The paper by Jaffee and Russell (1997) discusses different aspects of catastrophe insurance covering private insurance market, reinsurance and ILSs. A study by Gibson, Habib, and Ziegler (2006) compares exchange-traded catastrophe instruments and reinsurance. The authors try to understand the reasons behind the failure of exchange-traded catastrophe securities given high capacity and liquidity of financial markets. They propose a model that describes the insurer's problem of choosing between reinsurance and transferring catastrophic risk to financial market. They do this by comparing the expected payoff of both options. The authors refer to relative success of the OTC catastrophe instruments compared to their exchange-traded counterparts. Transactions of the former type of securities have been in the form of private placements and with a small number of qualified investors. This limits the presence of noise traders. They conclude that catastrophic risk has to be better understood if buyers are not restricted only to a small group of institutional investors. Hardle and Cabrera (2007) examine calibration of a parametric CAT bond for Mexican earthquakes. The authors refer to a mix of reinsurance and a CAT bond as a better and cheaper coverage for a ceding company (government in this case) compared to reinsurance itself. Finally, catastrophe insurance issues are also examined in the paper by Chakravarty and Kelsey (2006). This papers explains an 'Act of God Clause' that is present in many insurance and incentive contracts by ambiguity-aversion.

The second strand of the literature focuses on the conceptual issues of terrorism risk securitization. Woo (2004) refers to risk ambiguity, moral hazard and basis risk as major concerns when implementing terrorism risk securitization. At the same time, he sees multiple-event-risk transactions as a good way of transferring terrorism risk to capital markets and gaining confidence of both investors and rating agencies. He refers to a workers' compensation bond that may cover, for example, Los Angeles earthquake and terrorism, but causes loss of principal only if a second event occurred[4].

The idea of terrorism CAT bond has been initially suggested by Kunreuther (2002). Since then it had been developed in many other studies (see OECD (2005), Nell and Richter (2004), CTRMP (2006)). Besides the idea of terrorism CAT bonds, the concept of 'terrorism futures' market is discussed in literature (see Ray (2003) and Hanson (2005)). It relates to the controversial idea of prediction markets (see Wolfers and Zitzewitz (2003), Wolfers and Zitzewitz (2004)) that serve as aggregators of relevant information to forecast, for example, election outcomes, project completion dates, gas demands etc. The main concern about 'terrorism futures' relates to a possible speculative trading through Internet accounts and incentives for terrorists to commit acts of terror. Despite negative public reaction, a recent paper by Hanson (2005) gives a more favorable view of the 'terrorism futures' and provides a detailed description of possible design issues/concerns with implementing and using these instruments. These are combinatorics (possible terrorist attack scenarios), manipulation ('bad' guys might manipulate prices, 'noise' trading), moral hazard ('bad' guys benefit on trading), hiding pricing (alarming the public with terrorism threat and revealing public awareness of this threat to terrorists), and decision selection bias (a misleading impression of what speculators know about decision-contingent esti-

[4]The issue of terrorism risk for workers' compensation insurance in the U.S. is particularly severe because almost all states require terrorism risk to be covered in every workers' compensation policy (OECD (2005)). Workers' compensation insurers are obliged to pay wage loss and medical benefits to workers injured on the job without regard to cause and without loss limit. Terrorism and reinsurance experts have conceived of plausible catastrophic terrorist events that generate workers' compensation loss of 90 bn USD or more (Journal (2004)).

mates when decision makers know more than speculators). The author concludes that none of the above mentioned concerns seem unsolvable. According to Hanson (2005) 'Where there is a political will to purchase this concept, 'terrorism futures' have a reasonable chance of helping us to deal with terrorism'.

The third strand of literature focuses on the possible ways to quantify and model terrorism risk. Woo (2002) discusses several challenges associated with terrorism risk modelling. The author refers to the difficulties in predicting terrorist hazard (frequency of attacks) and their vulnerability (loss inflicted given a particular scenario of terrorist event). He focuses on strategic aspects of terrorism risk and possible architecture of terrorist networks (swarm intelligence). The author suggests an event-tree approach for estimation the success of planned attacks. Some literature focuses on application of the game theory in modelling of terrorism risk (see Woo (2002), Reactions (2002)). Given that this risk reflects rational planning and human intent, this theory can be a powerful tool to capture the human element of terrorist attacks. In his article, Major (2002) shows a possible application of the game theory to develop a probability distribution of losses[5].

Finally, some studies suggest hidden Markov models for implementing stochastic modelling of terrorist attacks. For example, according to Woo (2003), the Markovian concept of a system state is considered to be well suited to the fluctuating dynamics of the terrorism phenomenon, with the need to periodically update the threat situation. Singh, Allanach, Pattipati, and Willett (2004) propose the Adaptive Safety Analysis and Monitoring (ASAM) system that allows to implement counter-terrorism analysis using hidden Markov models and Bayesian networks. The authors illustrate the way the ASAM system works for two hypothetical models of terrorist activities at the Athens 2004 Olympics.

3.3 Securitization of Catastrophic Risks

While issuance of risk-linked securities such as CAT bonds and CAT futures has played an important part in securitization of the risk of natural disasters, terrorism risk transfer in a similar way has become a real challenge. Given that securitization in relation to natural

[5] Another alternative way to calculate the probabilities, the Delphi method, is used in the true probabilistic model developed by the Applied Insurance Research (Reactions (2002)).

catastrophes is well described in the literature (see Louberge, Kellezi, and Gilli (1999), Litzenberger, Beaglehole, and Reynolds (1996), Dubinsky and Laster (2004) and Jaffee and Russell (1997)), below we discuss conceptual issues of terrorism risk securitization only.

Terrorism risk is embodied within a worker's compensation risk, mortality risk, commercial/industrial property and business interruption risk, and event cancellation risk. The drastic reduction in terrorism risk (re)insurance cover after the 9/11 attacks has given a rise to a search for alternative risk transfer mechanisms.

There were two transactions that involved securitization of terrorism-related losses: the FIFA transaction, Golden Goal Finance Ltd., initiated by the FIFA governing body in September 2003 and the Vita Capital transaction implemented by Swiss Re in December 2003. In the first case, the CAT bond was aimed at covering revenue losses that would arise in the event of cancelation of the 18th FIFA World Cup scheduled to be held in the summer of 2006 in Germany. This transaction transferred the risk of the sporting event being canceled due to natural/ man-made catastrophes and terrorist events (OECD (2005)). In the second case, the extreme mortality risk was hedged against by means of the catastrophe-indexed notes. These instruments were linked to a rise in the constructed annual mortality index from natural disasters, epidemics, war and terrorist attacks (Brauner and Galey (2003)). While terrorism risk in these transactions was not the main risk under coverage, the over-subscription for these instruments demonstrated that investors were prepared to buy bonds with default tied to some level of this risk.

According to the OECD (2005), securitization of terrorism risk by means of CAT bonds with *multiple-event* and *multiple-risk* structure[6] appears to be an important condition of success: future securitization might try to mix-and-match different risks in order to dilute the terrorism component and to make the loss of principal contingent on the occurrence of two or more triggering events. While these financial instruments possess characteristics of standard CAT bonds, they also offer some other advantages because of their multiple-

[6]There have been some experience of multiple-event securitization in the area of natural catastrophes. For example, Phoenix Quake Wind covered second event Japanese earthquake and typhoon. It was issued on behalf of Zenkyoren, the Japanese National Mutual Insurance Federation of Agricultural Cooperatives. This securitization was rated Baa3 by Moodys, and BBB+ by S&P. The annual expected loss of this bond was 22 basis points. The coupon spread above LIBOR was 245 basis points. For more examples see Woo (2004).

event structure.

First, a multiple-event CAT bond is less likely to experience losses since it would require several and separate triggering events to occur before investors lose the principal. Second, the probability that several triggering events would occur is often less than that of one triggering event and such a multiple-event structure may reduce concerns about reliability of estimates for separate events. In other words, multiple-event contingency helps to mitigate the ambiguity that surrounds the risk of a single event. This consequently would result in a higher chance for a bond having obtained an investment-grade rating that would increase the number of institutional investors who would purchase this instrument, making securitization more viable[7]. Third, if one of the triggering events occurred, there would be time for the bond to be traded or to be put on watch list by a rating agency. Fourth, since the sequence of several triggering events may negatively affect an insurance company's credit rating and make the company insolvent, the multiple-event bond can provide protection against such contingency. It should be noted that although conceptually discussed, multiple-event CAT bonds have not been developed much. This paper is the first one to address this issue from a theoretical and numerical point of view.

To conclude, companies that decide to issue multiple-event CAT bonds would transfer catastrophic risks to the capital market and, as a result, may need to purchase less reinsurance[8] or, alternatively, to hold less capital to cover extreme losses. Among possible issuing companies are institutions that are the most exposed to these risks, namely, insurance and reinsurance businesses. In addition, governments, energy companies and oil producers may find these instruments attractive. For example, a multiple-event CAT bond that covers the risk of quakes and terrorist events in Mexico can be developed. Given that this country is one of the biggest oil producers and its exposure to earthquakes and terrorist events, the issuance of such bond may interest the government, oil companies and reinsurance businesses[9].

[7] For example, Moody's investment grade rating of FIFA transaction facilitated successful placement of the issue to the capital markets (Woo (2004)).

[8] Compared to reinsurance, CAT bonds can offer attractive pricing over multi-year term, systematic claim recovery, and in some cases, greater material security in the form of lower counterparty credit risk (Dubinsky and Laster (2004)).

[9] Terrorism risk in Mexico is related to the US-Mexico cross-borders (on the land and in the air), presence of foreign terrorism and criminal organizations.

3.3.1 Securitization Framework

The simple structure of a securitization procedure via the CAT bond is presented in the Figure 1 in Appendix A. The hedger/sponsor/ceding company (for example, an insurance company) pays a premium in exchange for a pre-specified coverage if a catastrophic event of a certain magnitude takes place and investors purchase a bond for cash. The total amount, premium and cash proceeds, are directed to a tailor-made fund, called a special-purpose vehicle company (SPV)[10], which issues the bond to investors (large institutional buyers[11]) and invests bond's proceeds (less transaction costs) in high quality securities portfolio, with low interest rate sensitivity. The securities portfolio is placed in a trust account as collateral for the debt service payments due on the bond. The SPV also engages into a fixed-floating interest rate swap that converts the interest returns on the portfolio into LIBOR based floating rate payments. Investors receive a relatively high spread above the LIBOR rate to compensate for the catastrophic risk exposure. Investors' cash flows namely coupons and/or principal are contingent on the catastrophe occurrence. In other words, a CAT bond functions like a fully collaterized multi-year reinsurance contract.

3.3.2 Diversification Issue

Standard CAT bonds provide attractive returns and portfolio risk diversification benefits for investors. The diversification argument relies on the fact that catastrophic risks are not or low correlated with market returns. If terrorism risk is among other catastrophic risk under coverage, diversification benefits of holding a CAT bond have to be revised.

Empirical evidence about the impact of terrorist attacks on the behavior of different financial markets shows both negative and positive reactions to these events (see results of the second paper of this thesis as well as papers by Chen and Siembs (2004), Johnston and Nedelescu (2005), Eldor and Melnick (2004) Carter and Simkins (2001), Karolyi and Martell (2005), Glaser and Weber (2004), Enders and Sandler (1991)). According to the existing research such industry sectors as insurance, airlines and travel/tourism exhibit

[10]The SPV is typically structured as a Cayman Islands or Bermuda exempt company whose common shares are held by a charitable trust. This structure shelters the SPV from a potential bankruptcy, and resembles the approach often adopted in asset-backed securitization. (Dubinsky and Laster (2004))

[11]To date, the majority of CAT bonds have been sold to investors pursuant to Rule 144A.

negative reaction to terrorist events. In contrast, the results of the second paper of this thesis show that aero/defense, pharma/biotech and oil/gas sectors may show a significant positive reaction. In addition, government bonds and the commodity/gold indices that are traditionally considered as ‘safe-haven’ investments, may also benefit from unsettling news about terrorist events.

In terms of the strength of the impact, terrorist attacks display a short-term rather than a long-term effect on the markets’ behavior (see second paper of this thesis and paper by Chen and Siembs (2004)). Consequently, because of the markets’ resilience to terrorist attacks, returns on the investor’s portfolio can recover fast.

If investors prefer strongly diversified portfolios and hold financial assets with returns that are positively correlated with the risk of terrorism (commodities, US government bonds, defense and pharma/biotech stocks) then possible losses on the CAT bond due to this risk may be offset by the gains on other assets. However, if among other assets are insurance, airline and travel/tourism stocks, then terrorist events will have a more pronounced negative effect on the portfolio performance.

3.4 The Model

The multiple-event CAT bond we are interested in, covers exposure to natural catastrophes and man-made disasters, including terrorism. From the prospective of insurer, these events can affect multiple lines of insurance such as life, commercial/industrial property, workers’ compensation, accident and health. In this study, we focus on catastrophic property losses and on catastrophic mortality. The cash flows of the bond are linked to two types of triggering events (see Figure 2). The first type of event is associated with either catastrophic property damage or catastrophic mortality that is above a corresponding attachment level. The second type of event is when both catastrophic property losses and deaths are above their respective attachment levels. The CAT bond is a coupon-paying instrument. Investors lose their coupon payments when one of the two triggering events happens. The principal is fully at risk if both triggering events occur.

We use indemnity trigger for property losses, which involves direct property losses of the ceding party due to catastrophic events. In indemnity deals, there is no basis risk. At the same time, there is a potential of a moral hazard problem. This is because the issuing party may have an incentive to settle claims more generously when losses approach the attachment level. We assume that evaluation of the incurred catastrophe property losses and the number of deaths respectively is implemented by outside institutions such as the ISO's Property Claim Services[12] and the Office of National Statistics[13]. Finally, we make a simplifying assumption that both the amount of property losses and the number of deaths are available at the end of the day of occurrence of a catastrophic event.

3.4.1 Assumptions

- We consider a coupon paying bond with maturity T years. The face value of the bond is F and coupon payments C_t are paid annually at times $t = 1, 2, \ldots, T$. In case of catastrophe events, both, coupons and the principal are at risk.
- The potential total daily loss amounts $I^L = (I_t^L)_{0 \leq t \leq T}$ are non-negative random variables that follow some distribution F^L. The aggregate property loss process up to time t is given as $I_t^{AL} = \sum_{i=0}^{t} I_i^L$, $t \in [0,T]$.
- The potential number of deaths per day $I^D = (I_t^D)_{0 \leq t \leq T}$ are random variables follow some distribution F^D.
- The daily property losses I_t^L and daily deaths I_t^D may be dependent. We denote the dependence coefficient as τ_{LD}.
- The attachment point for property losses is given as a threshold total annual loss amount. The initial value of the attachment point is set at time $t = 0$ and equal to L_{AP}. The initial value has to be changed at the beginning of each year during the life of the bond (losses are accumulated on daily basis over the whole life of the bond): $I_{AP_t}^L = \sum_{i=0}^{t-1} I_i^L + L^{AP}$, $t = 1, 2, \ldots, T$, where $I_{AP_t}^L$ is the value of attachment point set for year t.

[12] ISO's Property Claim Services (PCS) unit is the internationally recognized authority on insured property losses from catastrophes. Check www.iso.com for more details. There is another institution called AIR. See www.air-worldwide.com.

[13] See www.statistics.gov.uk for UK, www.cdc.gov/nchs/ for US, www.bfs.admin.ch for Switzerland.

- The attachment point for deaths is defined as a threshold number of deaths per day I^{D}_{AP}. It stays at this level for the whole life of the bond.
- Triggering events are defined in terms of the following threshold times:

  $\tau_1 = \inf\{t > 0 : \mathrm{I}^{AL}_t > \mathrm{I}^{L}_{AP_s}\}$, where s=[t]+1 if t < T and s=T if t=T.

  $\tau_2 = \inf\{t > 0 : \mathrm{I}^{D}_t > \mathrm{I}^{D}_{AP}\}$,

  $\tau_3 = \min\{\tau_1, \tau_2\}$,

  $\tau_4 = \max\{\tau_1, \tau_2\}$.
- One-period interest rates are $\{r(k) \mid k = 0, 1, 2, \ldots, T-1\}$.

3.4.2 Pricing Framework

Pricing of derivative securities in complete market setting involves replicating portfolios. This way of pricing, however, does not work for the multiple-event CAT bond under our consideration. The bond derives its value based on the level of annual aggregate catastrophe property losses and daily catastrophe mortality, but we have no efficiently traded mortality and catastrophic loss indices with which to create a replicating hedge. Situations like this implies incompleteness of the market. In this study, we handle market incompleteness (existence of many equivalent martingale probability measures) by means of the representative agent pricing model (see Huang and R.Litzenberger (1988), Karatzas (1997)).

We assume that financial market variables are modelled on the filtered probability space $(\Omega^{(1)}, \mathscr{P}^{(1)}, \mathbb{P}_1)$. The relevant financial market variables when valuing CAT bonds are, for example, the term structure of interest rates. The catastrophic risk variables are modelled on the filtered probability space $(\Omega^{(2)}, \mathscr{P}^{(2)}, \mathbb{P}_2)$. The probability measure $\mathbb{P}_2$ is a physical probability measure governing catastrophe events. The probability space for the full model is given as $(\Omega, \mathscr{P}, \mathbb{P})$, where $\Omega := \Omega^{(1)} \times \Omega^{(2)}$; $\mathscr{P}_k = \mathscr{P}^{(1)}_k \times \mathscr{P}^{(2)}_k$ for $k = 0, 1, \ldots, T$; $\mathbb{P}(\omega) = \mathbb{P}_1(\omega^{(1)})\mathbb{P}_2(\omega^{(2)})$, where $\omega = (\omega^{(1)}, \omega^{(2)})$ is a generic state of the world describing the state of the financial market variables and the catastrophic risk variables. *It should be noted that this setting implies independence of events that depend only on economic risk variables and those that depend only on catastrophic risk variables.*

The cash flows to the bondholder can be described in the following way:

$$CF(k) = \begin{cases} \mathrm{C}I_{\tau_3>k} & k = 1, 2, \ldots, \text{T-1}; \\ \mathrm{C}I_{\tau_3>k} + \mathrm{F}I_{\tau_4>k} & k = \text{T}. \end{cases}$$

To price this stream of cash flows in the incomplete market we apply the representative agent pricing model (RAPM). According to this model, the price $P_n(CF)$ of a generic future cash flow process $CF = \{CF(k) \mid k = 1, 2, \ldots, T\}$ at time 0 is given by the expectation

$$P_0(CF) = E_{\mathbb{P}}\left[\sum_{k=1}^{T} \frac{u'_k(C^*(\omega, k))}{u'_0(C^*(0))} CF(k)\right], \tag{3.1}$$

where $u_k(C^*(\omega, k))$ is the utility of the amount of the consumption good endowed to the entire economy in state ω at time k. Note that the form of the utility function and the aggregate consumption $C^*(w, k)$ is removed from the pricing analysis later on. This is done by relating the representative agent valuation formula to the valuation measure approach of arbitrage-free pricing.

The one-period interest rates are defined as

$$\frac{1}{1 + r(k)} := \frac{1}{u'_k(C^*(\omega, k))} E_{\mathbb{P}}[u'_{k+1}(C^*(\omega, k+1)) \mid \mathscr{P}_k], \tag{3.2}$$

for $k = 0, 1, 2, \ldots, T-1$ (see Appendix C).

Under the assumptions of the RAPM, the price can be equivalently obtained as a discounted expectation under the probability measure $\mathbb{Q}$:

$$P_0(CF) = E_{\mathbb{Q}}\left[\sum_{k=1}^{T} \frac{1}{[1 + r(0)][1 + r(1)] \ldots [1 + r(k-1)]} CF(k)\right], \tag{3.3}$$

where the Radon-Nikodym derivative for all information over $[0, T]$ is given as

$$\frac{d\mathbb{Q}}{d\mathbb{P}} \mid_{\mathscr{P}_T} (\omega) := [1 + r(0)][1 + r(\omega, 1)] \ldots [1 + r(\omega, T-1)] \frac{u'_T(C^*(\omega, T))}{u'_0(C^*(\omega, 0))}. \tag{3.4}$$

A detailed explanation about the change of measure, covering the proof that the process $\zeta(T) = \frac{d\mathbb{Q}}{d\mathbb{P}} \mid_{\mathscr{P}_T}$ is a $\mathbb{P}$-martingale on the filtration $\mathscr{P}$, can be found in Cox and Pedersen (2000). Equivalence of formulas (3.1) and (3.3) implies that *the knowledge of one-period interest rates and the risk-neutral valuation measure* $\mathbb{Q}$ *is equivalent to knowledge of the representative investor's utility function and the aggregate consumption process.*

We *assume that the aggregate consumption depends only on the financial risk variables.* This implies that the Radon-Nikodym derivative in (3.4) also depends only on financial risk variables. *Under the valuation measure $\mathbb{Q}$ those events that depend only on financial risk variables are independent of those events that depend only on catastrophic risk variables.* Assuming that cash flows $CF(k)$ on CAT bond depend only on the catastrophic risk variables, this has important implication on the pricing of such bond:

$$P_0(CF) = \sum_{k=1}^{T} P(0,k)E_{\mathbb{P}}[CF(k)] = \sum_{k=1}^{T} P(0,k)E_{\mathbb{P}_2}[CF(k)], \tag{3.5}$$

where

$$P(0,k) = E_{\mathbb{Q}}\left[\frac{1}{[1+r(0)][1+r(1)]\dots[1+r(k-1)]}\right] \tag{3.6}$$

stands for a price at time t=0 of a non-defaultable zero-coupon bond with face value 1 maturing at time k (see Appendix D for more details about this result). The price of the bond at time $t=0$ is given by

$$\mathrm{P}_0^B(CF) = \sum_{t\in\{1,\dots,T\}} \mathrm{C}P(0,t)E_{\mathbb{P}_2}[\mathbb{I}_{\tau_3>t}] + \mathrm{F}P(0,T)E_{\mathbb{P}_2}[\mathbb{I}_{\tau_4>T}]. \tag{3.7}$$

We simplified this formula (see Appendix E) to

$$\mathrm{P}_0^B(CF) = \sum_{t\in\{1,\dots,T\}} \mathrm{C}P(0,t)\mathbb{P}_2\left[\sum_{i=0}^{t} I_i^L \le I_{AP_t}^L, \sup_{\{0\le j\le t\}} I_j^D \le I_{AP}^D\right] +$$
$$\mathrm{F}P(0,T)[1-\mathbb{P}_2(\tau_1\le\tau_2 \mid \tau_2\le T)\mathbb{P}_2(\tau_2\le T) - \mathbb{P}_2(\tau_2\le\tau_1 \mid \tau_1\le T)\mathbb{P}_2(\tau_1\le T)].$$

We do not expect a close-form solution for such a complex contingent security and thus we estimate the price of the bond numerically.

3.4.3 Numerical Analysis

This section estimates the price of a multiple-event CAT bond using historical data on the UK catastrophe property losses and deaths provided by Swiss Re (see Table 1 in Appendix B). The UK was exposed to the risk of natural catastrophes and terrorist events in the past[14]. According to experts' forecast, exposure of the UK to these catastrophe events will not diminish in the future. Therefore transferring catastrophe risks to capital markets by means of CAT bonds can be an attractive securitization strategy for insurance and reinsurance in this country.

Data Calibration

The data cover daily property losses and deaths due to natural and man-maid disasters that occurred between 1970 and 2005 (see Figures 3-4 in Appendix A). In order to calibrate our model we have to estimate a distribution of the time of occurrence of catastrophe events as well as a joint distribution of catastrophe property losses and deaths.

Time of occurrence of catastrophic events

When implementing modelling of the time of occurrence of catastrophe events we look at their inter-arrival times. We see that data exhibit autocorrelation and are not random (see Figure 5 (a) in and a corresponding value of the Ljung-Box test of randomness). We implement a log-transformation of the inter-arrival times and see that data become random (see Figure 5 (b)-(d)). Next, we graphically examine the data transformed. We see that data are asymmetrical and have heavy tails (see Figure 6). To capture these characteristics, we consider a skewed student's t-distribution (with parameters η and λ as it is described in Hansen (1994)) as a possible analytical distribution of the normalized log-transformed inter-arrival times. We estimate parameters of this distribution using the MLE technique and check the adequacy of the fit graphically and based on non-parametric tests. Our analysis confirms that a skewed student's t distribution with $\eta = 10.7176$, $\lambda = -0.3380$ describes the log-transformed data well both based on the graphical examination (see Figures 7-8) and according to the test statistics (the Kolmogorov-Smirnov

[14] Hurricane force winds in Oct. 1987 caused extensive damage of $1,957$m GBP. Storms and flooding in January-February 1990 resulted in $3,158$m GBP. Catastrophic "extreme rainfall events" in summer 2007 left more than a third of a million people without drinking water, nearly 50,000 people without power, thousands more people homeless and caused more than 2bn GBP worth of damage (McCarthy (2007)). As to exposure to terrorism risk, London bombings on 7th of July, 2005 caused 56 deaths, about 700 people injured, and 51.5 m USD in insured property loss. Among more recent events in London were two car bombs that were discovered and disabled before they could be detonated on the 29th of June, 2007 .

test statistics at the 1% significance level is 0.05 and p-value is 0.97). Finally, we check the goodness of fit by comparing empirical inter-arrival times with simulated inter-arrival times (see Figure 9) and see that the fit is adequate (the Kolmogorov-Smirnov test statistics at the 1% significance level is 0.04 and p-value is 0.98).

Marginal Distributions of Catastrophe Deaths and Losses

We consider the data on deaths and losses at times of occurrence of events. The data include zero values in the cases when a catastrophe event has caused either property losses or deaths. To capture the presence of zeros, we model margins using a distribution of the following form:

$$f(x) = \begin{cases} \delta_0 & \text{with probability } (1-w); \\ f^*(x) & \text{with probability } w, \end{cases}$$

where δ_0 is a Dirac distribution, $f^*(x)$ describes the distribution of non-zero values and w is a percentage of such values. We define $f^*(x) = f^*_D(x)$ and $f^*(x) = f^*_L(x)$ as density functions of deaths and losses correspondingly.

We consider Poisson and Negative Binomial distributions as possible candidates to describe $f^*_D(x)$ (see Figure 10 (a)). While the Poisson distribution does not fit the data well[15], the Negative Binomial distribution with parameters $r = 0.82$ $p = 0.03$ passes all the tests (the Kolmogorov-Smirnov test statistics at the 1% significance level is 0.08 and p-value is 0.84, for graphical examination see Figure 10 (b)). Next, we check how well the proposed distribution $f(x)$ with $f^*_D(x) \sim \text{NB}(0.82, 0.03)$ describes the data on catastrophe deaths. We see that graphically (Figure 11) and based on the test statistics (the Kolmogorov-Smirnov test statistics at the 1% significance level is 0.04 and p-value is 0.99) the fit is adequate.

In the case of losses, we find that the Skew-Normal distribution with a location parameter $\xi = 3.4342$, a scale parameter $\omega = 1.7197$ and a shape parameter $\alpha = 2.9088$ provides a good fit to the log-transformed non-zero losses[16] (the Kolmogorov-Smirnov test

[15] The Kolmogorov-Smirnov test at the 1% significance level is 0.51 and p-value is 0.00.

[16] We also check the Lognormal and Pareto distributions as possible candidates to describe the data on losses.

statistics at the 1% significance level is 0.08 and p-value is 0.54, for a graphical examination see Figure 12). Finally, we analyze the adequacy of the fit of $f(x)$ with $f_L^*(x) \sim$ SN$(3.4342, 1.7197, 2.9088)$ to the data on catastrophe property losses. We find that graphically (see Figure 13) as well as based on the test statistics (the Kolmogorov-Smirnov test statistics at the 1% significance level is 0.05 and p-value is 0.90.) the fit is good.

Joint Distribution of Catastrophe Deaths and Losses

To model a joint distribution of losses and deaths, we apply a concept of copula (see Nelsen (1999), Joe (1997), Mendes (2004), Malevergne and Sornette (2001), Cherubini and Luciano (2001)). To assure uniqueness of a copula and appropriateness of concordance measures for a copula's calibration, we use continued versions of the data as it is suggested in Denuit and Lambert (2005). We look at Archimedian class of copulas and use the Kendall's tau dependence measure[17]. A preliminary examination of dependence by means of a scatter plot shows that losses and deaths exhibit a low dependence (see Figure 14). Note that the dependence affects the probability of the bond's default and, therefore, its price. When calibrating parameters of different copulas, we rely on the relationship between a copula parameter θ and computed value of the Kendall's tau: $\tau_{LD} = -0.1654$, $\theta_C = -0.2838$ for the Clayton copula and $\theta_F = -1.5221$ for the Frank copula[18]. We check the fit of selected copulas to the data non-parametrically, using a concept of cardinality. In other words, we compare Archimedian copulas under consideration with their empirical equivalent. The copula fit test is performed using the algorithm presented in Genest and Rivest (1993). We look at a random variable $V = F(I^L, I^D)$ with a joint distribution

However, these distributions do not pass the goodness of fit tests. The Anderson-Darling test at 1% significance level for Lognormal distribution gives a p-value of 0.0001. The Kolmogorov-Smirnov test at the same significance level for Pareto distribution is 0.16, p-value is 0.02.

[17] Kendall's tau is defined as the probability of concordance minus the probability of discordance for the pairs of iid random vectors (**?**):

$$\rho_\tau(X_1, X_2) = P((X_1 - \tilde{X}_1)(X_2 - \tilde{X}_2) > 0) - P((X_1 - \tilde{X}_1)(X_2 - \tilde{X}_2) < 0).$$

[18] We do not use the Gumbel copula because it requires $\tau > 0$.

function $K(v)$. For Archimedian copulas,

$$K(v) = v - \frac{\phi_\theta(v)}{\phi'_\theta(v)},$$

where $\phi_\theta(v)$ is a generator of a copula. The empirical joint distribution function $K_N(v)$ is computed as

$$K_N(v) = \frac{\Sigma_i \delta(v - Z_i)}{N},$$

where δ is a Dirac function and Z_i is defined as

$$Z_i = \frac{\sharp\{(I_j^L, I_j^D) : I_j^L < I_i^L, I_j^D < I_i^L\}}{N-1},$$

with $\sharp$ standing for a cardinality of the set $\{\cdot\}$.

A graphical examination of the fit (see Figure 15-16) as well as the analysis of mean-square errors shows that the Clayton copula[19] provides a better fit to the data than the Frank copula (the corresponding mean square errors are 0.009 for the Clayton and 0.0022 for the Frank copulas). Although historical data exhibit low negative dependence between losses and deaths, the Clayton copula, in general, allows to generate data with a positive dependence as well. We use this property when modelling positively dependent catastrophe property losses and deaths in our scenario analysis. To conclude, the joint distribution of catastrophe property losses and deaths described by the Clayton copula is given as

$$\Pr(I_D \leq x, I_L \leq y) = C(u_1, u_2) = \left(u_1^{-\theta_C} + u_2^{-\theta_C} - 1\right)^{-1/\theta_C}, \tag{3.8}$$

where $u_1 = F^D$, $u_2 = F^L$ and $\theta_C = -0.2838$.

Dynamics of the CAT Bond Price

We assume that the bond has a maturity of 3 years, pays annual coupon rates of 300 bps over a three-month LIBOR of 6% and has a principal of 100 USD. We set a one-period interest rate r to be equivalent to the LIBOR rate and assume that it stays constant during the life of the bond. We choose attachment points based on the values of 80% quantiles of the distribution of historical catastrophe deaths and annual property losses respectively:

[19]The generator function of the Clayton copula is given as $\phi_{\theta_C}(v) = \frac{1}{\theta_C}\left(v^{-\theta_C} - 1\right)$.

$I_{AP}^{D} = 48$ and $L_{AP} = 1362.70$ m. USD. This set of parameters corresponds to our base case. In addition, we implement sensitivity analysis of the bond's price to different values of attachment levels, dependence and time to maturity. Finally, we compute the price of a zero-coupon CAT bond in each case.

We implement 1000 Monte Carlo simulations and obtain the following results (see Table 2 in Appendix B). In the base case (Case I), the price of the coupon paying multiple-event CAT bond is 98.57 USD. As expected, in all cases considered, a price of zero-coupon CAT bond is smaller than a corresponding price of a coupon paying CAT bond. We analyze the sensitivity of a bond's price to the time to maturity. We find an inverse relationship between price of the bond and its time to expiration (Case II and III). A possible explanation of this result may relate to the fact that over a shorter period of time, the probability that catastrophe event would cause property losses and deaths above their attachment points is lower than over a longer period of time. This implies a smaller probability of the bond's default and, therefore, a higher price. However, this relationship may not always work for *coupon* paying CAT bonds if their maturities are extended (Catastrophe Event A, Table 3). This is because investors get more coupons over a longer period of time and unless a triggering event occurs, this consequently results in the increase of the bond price. This effect is especially pronounced in the case of high attachment levels that lead to reduction in the probability of losing cash flows on the bond and its higher price (Case VII). A high level of positive dependence between catastrophe losses and deaths increases the probability of reaching the attachment levels and therefore the probability of a bond's default. Thus, by increasing the dependence we expect the price of the bond to decrease. We confirm this numerically in the Case IV. When dependence is increased to 0.80, a smaller price (compared to the base case) of 97.07 USD is obtained. Note, that in the base case, historical data exhibit the Kendall's tau dependence of -0.16. This negative dependence can be explained by the fact that catastrophic events often cause either property losses or deaths and few events result in both types of casualties.

Next, we evaluate the price of the CAT bond assuming possible independence of catastrophe losses and deaths (Case V). Computed results confirm our finding that a price of the bond decreases (increases) as a positive (negative) dependence between catastrophe losses and deaths gets stronger. Comparison of CAT bond prices in the Case III and IV shows that the price is more sensitive to changes in the time to maturity than to changes in the level of dependence. In both cases, the price is expected to fall due to increased probability of default. However, the decrease is more pronounced in the case of extended maturity.

Next, we analyze the reaction of the bond's price to increases in attachment (threshold) levels. In Case VI, we change attachment levels to the values of 99% quantiles of the distribution of historical catastrophe deaths and annual property losses respectively. As expected, we observe increase in the price of the bond. This is because higher attachment points imply a lower probability of default and therefore a higher price. Case VII looks at the bond's price when, in addition to increased thresholds, the time to maturity is extended to 4 years. This case is interesting since on the one hand high threshold levels drive the price up (lower probability of default) and on the other hand longer time to maturity pushes the price down (higher probability of default). We see that for a coupon paying CAT bond the impact of increase in attachment levels dominates the effect of extended maturity. As a result, the price of coupon-paying CAT bond in the Case VII is higher than in the Case VI (108.90 USD versus 107.03 USD). The opposite effect is observed for a zero-coupon CAT bond, which price declines from 83.96 USD to 79.21 USD.

In addition, we evaluate the price of the bond for a scenario of extreme events of the magnitude of the 9/11 attacks (Catastrophe Event A in Table 3) and higher, as it can be in the case of chemical, biological, radiological and nuclear terrorism incidents[20] (Catastrophe Event B in Table 3). Note that the level of property losses due to the 9/11 attacks is close to the property damage of 40 bn USD predicted due to possible flooding in the UK because of the climate change[21] (see ABI (2005)). Estimated price of the coupon paying CAT bond in the case of Catastrophe Event A reaches 108.02 USD. As expected, this price is higher than in the base case because of the reduced probability of the bond's default. Note that price stays at the same level when dependence is increased to 0.80.

[20]The calibration of a threshold level for property losses for this event is implemented using the forecast by Insurance Information Institute for major US cities (see Valverde and Hartwig (2006))

[21]The 9/11 attacks resulted in almost 3000 deaths and 35.6 bn USD in insured loss.

However, it increases for a coupon paying CAT bond and decreases for a zero-coupon CAT bond when, in addition, the maturity is extended to 4 years. This result confirms a more responsive behavior of the bond's price to changes in the time to maturity compared to changes in dependence. Similar results are obtained for the Catastrophe Event B.

Finally, we see that despite the increase in the thresholds and/or in the dependence level, the price of a zero-coupon CAT bond stays at the level of 83.96 USD for T=3 and 79.21 USD for T=4 starting from the Case VI. This implies that when threshold levels reach certain values, the probability of losing the principal becomes extremely low and it is not affected by changes in the dependence. In such cases, the price of a zero-coupon CAT bond stabilizes at the level of $1/(1+r)^T$. We analyze the price stabilization issue in more detail in Table 4 and Table 5. By varying the attachment level of catastrophe losses and keeping the attachment level of deaths fixed, we find a level of catastrophe losses at which CAT bond prices do not change. Table 4 presents CAT bond prices when the attachment level of deaths is set to 48 people. Table 5 describes CAT bond prices when this level is equal to 3000 people. In both cases, when the attachment level of catastrophe losses reaches 25 bn USD, prices of coupon-paying CAT bonds become stable and reach 104.78 USD and 108.02 USD respectively. Figure 17 shows graphically the results of our computations. It is clear that for attachment levels of deaths that are in the interval $(48, 3000)$ the values of CAT bond prices would lie between the two lines in Figure 16. In relation to prices of zero-coupon CAT bonds and for attachment mortality level of 48 people, prices stabilize at the level of 83.96 USD when attachment level of catastrophe losses is 17.8 bn. USD. However, for a higher attachment mortality level of 3000 people the prices of zero-coupon CAT bonds stay at 83.96 USD independently from the attachment level of property losses. These results are interesting to look at from investors' point of view. We observe that starting from a certain attachment level of property losses that is much above the 99.9% quantile of their historical distribution, investors have a choice to invest in multiple-event CAT bonds with different attachment levels of property losses (keeping attachment point of deaths constant) and to pay the same price. This phenomenon is due to extremely low probabilities of default of such bonds. At the same time, investors' choice is driven by their risk-return preferences and investors with more risk appetite most probably will not exploit this price stabilization phenomenon. Instead, they may prefer to invest in cheaper multiple-event CAT bonds with significantly higher default probabilities and higher returns.

3.5 Conclusions

This study is the first of its kind to develop a model for pricing of multiple-event coupon paying CAT bonds covering exposure to catastrophic risks, including terrorism. The pricing of the bond is implemented under the assumption of market incompleteness by relying on the representative agent pricing model.

In contrast to the existing literature on pricing of standard CAT bonds, the payoffs on the bond in our model are linked to two types of underlying processes: insured catastrophe property losses and catastrophe daily deaths. In addition, the bond under consideration has a multiple-event and multiple-risk structure. We assume that both, the bond's coupons and its principal are at risk if the triggering events occur. This model views catastrophic terrorism risk as one of the main catastrophic risks effecting the cash flows of the bond.

An important contribution of this work is that it provides a numerical evaluation of the price of the bond under consideration. We implement Monte Carlo simulations of the bond price using the UK catastrophe data provided by Swiss Re. We consider different catastrophe scenarios and compute corresponding prices of coupon-paying and zero-coupon CAT bonds. These scenarios include events of the magnitude of the 9/11 attacks and higher as may be the case as a result of chemical, biological, radiological and nuclear terrorist incidents.

The results of this study indicate that the price of the bond has a direct relationship with threshold levels, but an inverse relationship with stronger positive dependence between losses and deaths. As to time to maturity, there is an inverse relationship between the price of the bond and its time to expiration. Although this relationship always works for a zero-coupon catastrophe bond, it may not always hold when a bond pays coupons. In general, we find a less responsive behavior of the bond's price to changes in dependence compared to changes in the time to maturity. In addition, we see that, for a coupon paying CAT bond, the impact of increase in attachment levels on the bond's price dominates the effect of extended maturity. The opposite result is observed for a zero-coupon CAT bond.

The model suggested in this paper may be interesting to insurance and reinsurance companies and other financial institutions. These organizations may want to consider using multiple-event CAT bonds to transfer to capital markets their exposure to catastrophic risks, including terrorism risk. With respect to the latter risk, our model allows to protect from losses and deaths that are of significantly lower magnitude than what current terrorism related risk transfer securities allow.

Appendix A: Figures

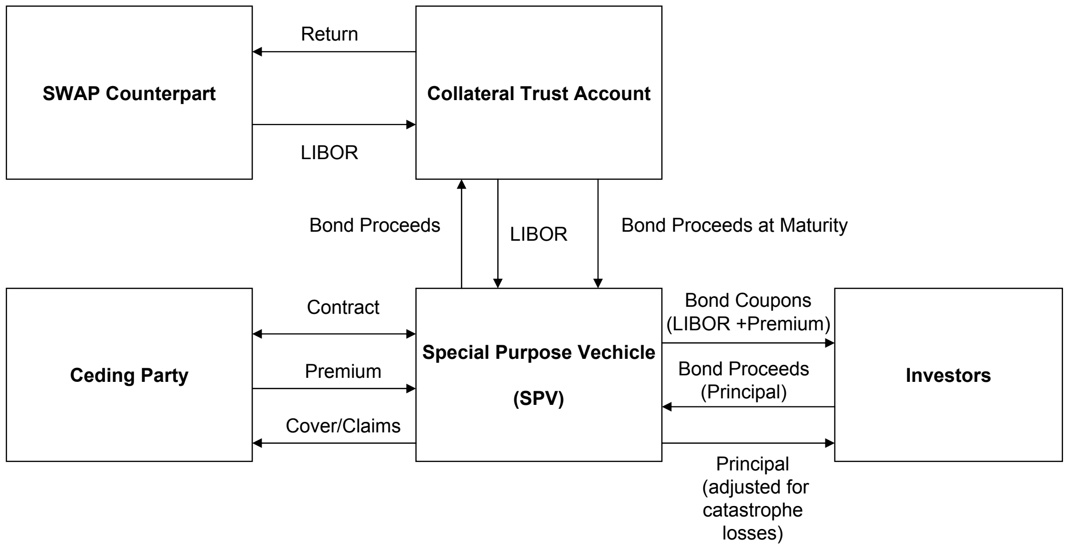


Figure 1. The CAT Bond Design. The figure shows a simple structure of a securitization via the CAT bond.

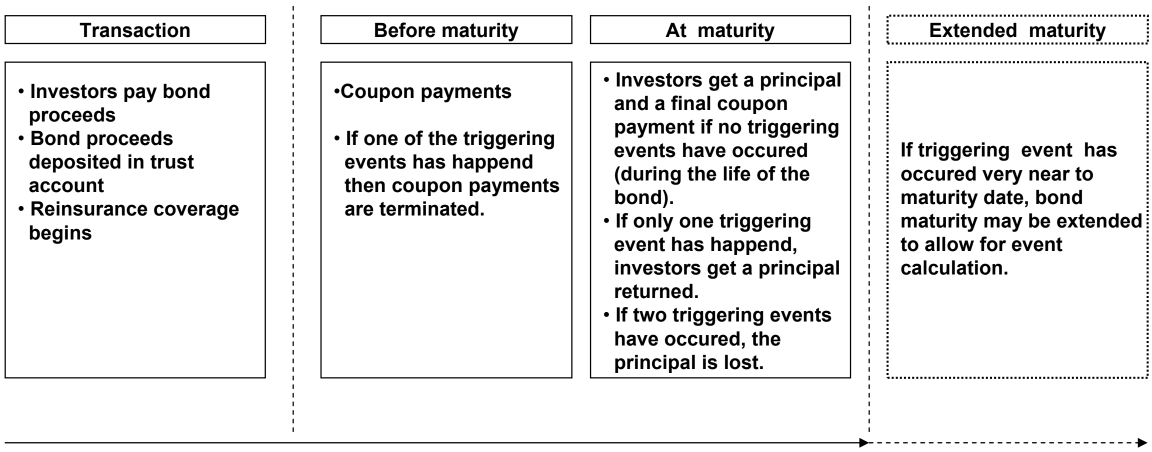


Figure 2. Timing of Cash Flows of the Multiple-Event CAT Bond.

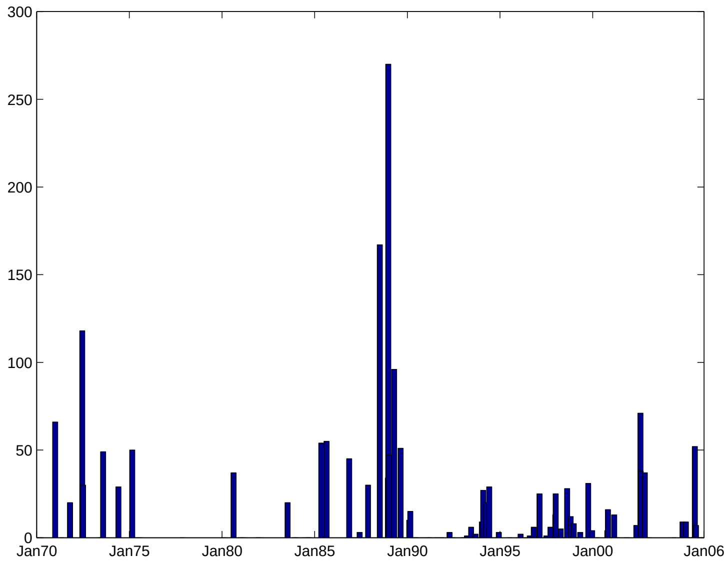


Figure 3. Catastrophe Deaths The figure shows daily number of catastrophe deaths in the UK from January 1970 to January 2006.

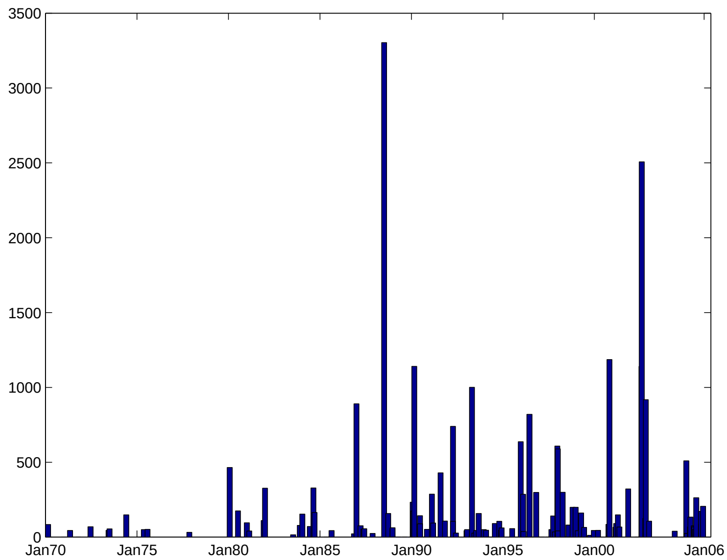


Figure 4. Catastrophe Insured Property Losses. The figure displays data on daily catastrophe property losses in USD m. in the UK from January 1970 to January 2006.

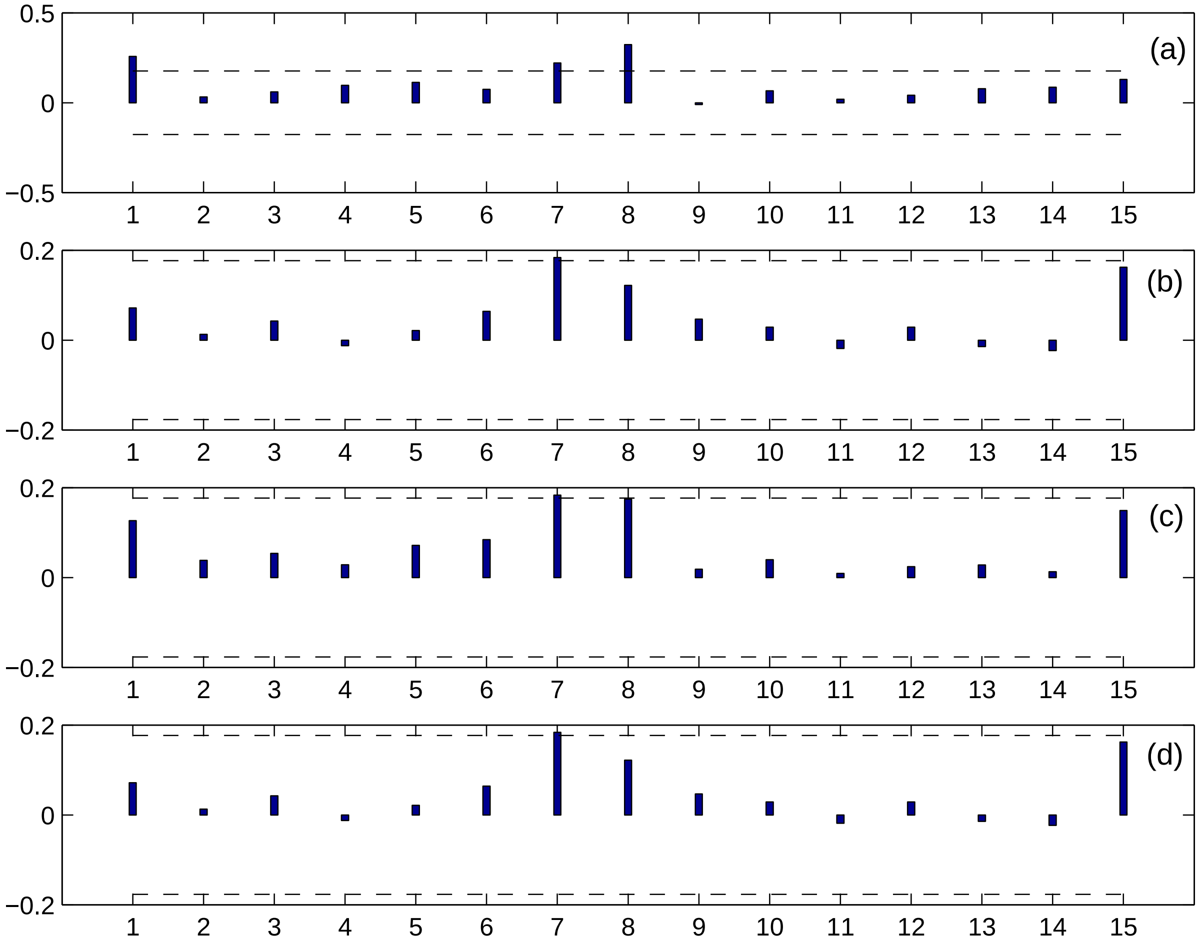


Figure 5. Sample Autocorrelation Function of (a) inter-arrival times; (b) log of inter-arrival times; (c) square of log of inter-arrival times; (d) absolute value of log of inter-arrival times. Ljung-Box Test for randomness at significance level $\alpha = 0.01$: (a) rejected (L-J test statistics is 47.72, p-value is zero); (b) accepted (L-J test statistics is 18.67, p-value is 0.23); (c) accepted (L-J test statistics is 25.58, p-value is 0.04); (d) accepted (L-J test statistics is 18.67, p-value is 0.23).

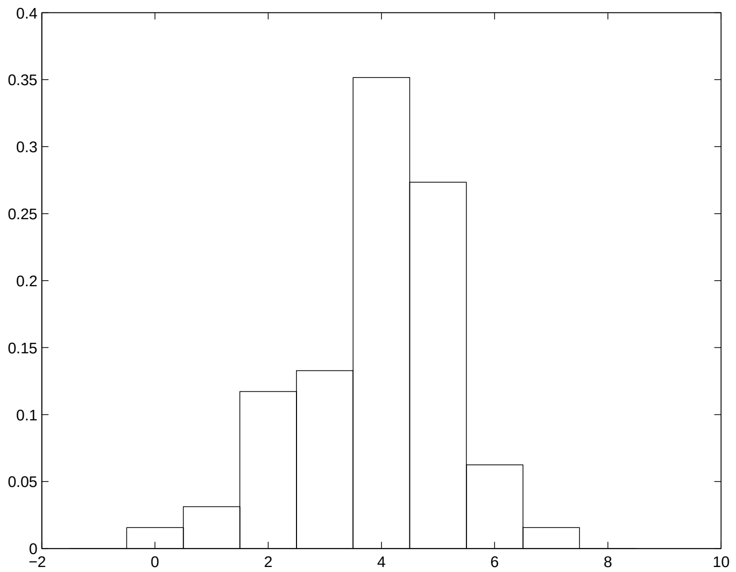


Figure 6. Histogram of the Log of Inter-Arrival Times of Catastrophe Events.

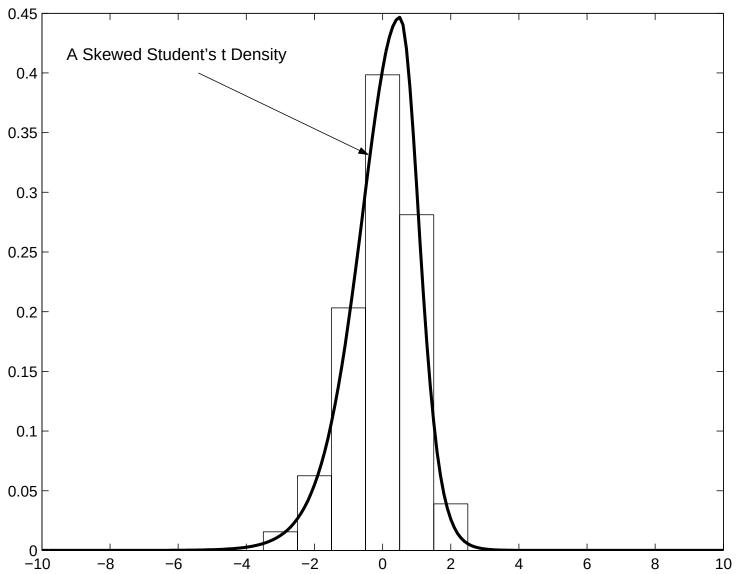


Figure 7. Histogram and Fitted Density of the Log of Inter-Arrival Times of Catastrophe Events. A skewed Student's t density with $\widehat{\eta} = 10.7176$, $\widehat{\lambda} = -0.3380$ is applied to standardized data on log of inter-arrival times.

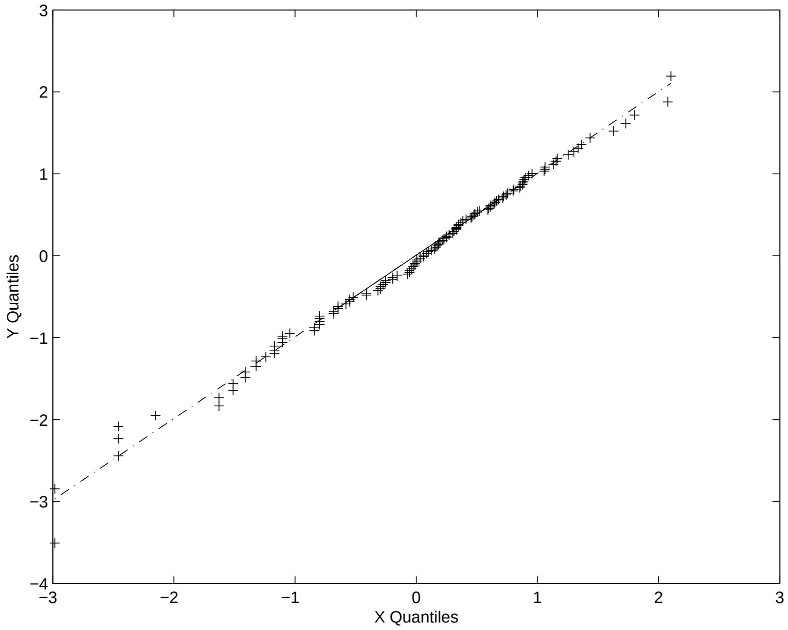


Figure 8. QQ Plot of the Log of Inter-Arrival Times. Actual data versus simulated.

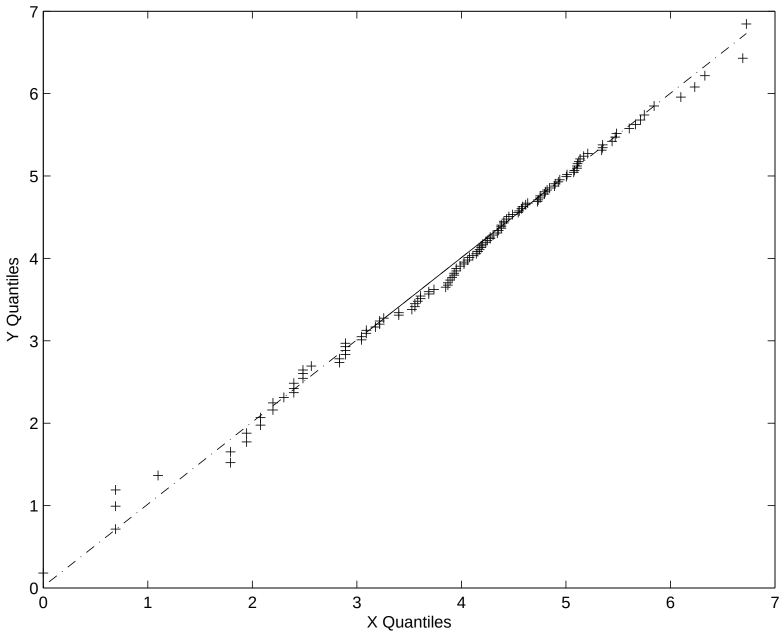


Figure 9. QQ Plot of Inter-Arrival Times of Catastrophe Events. Actual data versus simulated.

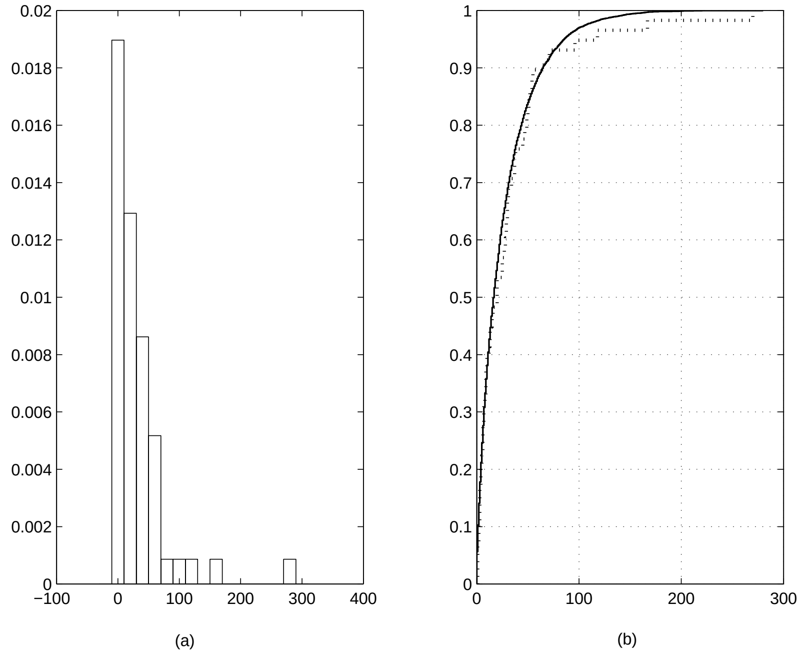


Figure 10. Non-Zero Catastrophe Deaths. (a) Histogram; (b) Estimated (solid) and Empirical (dotted) Cumulative Distribution Function.

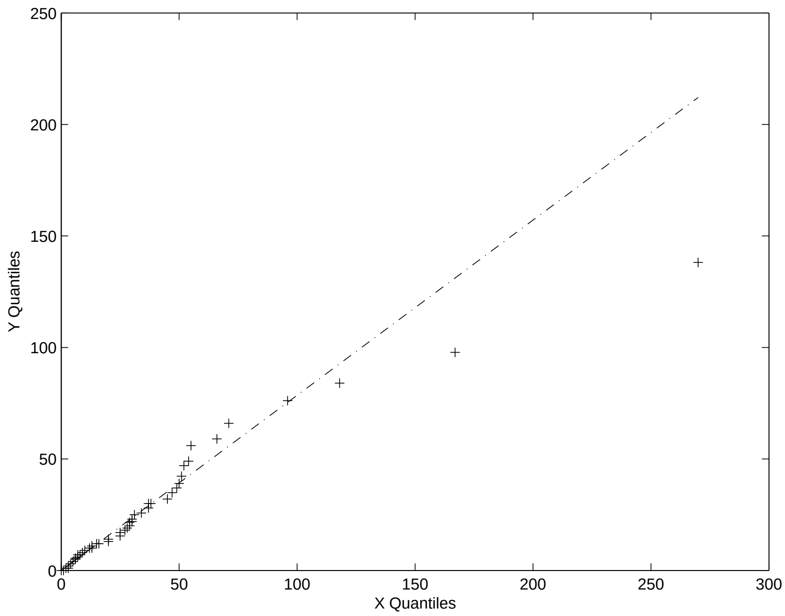


Figure 11. QQ Plot of Catastrophe Deaths. Actual data versus simulated.

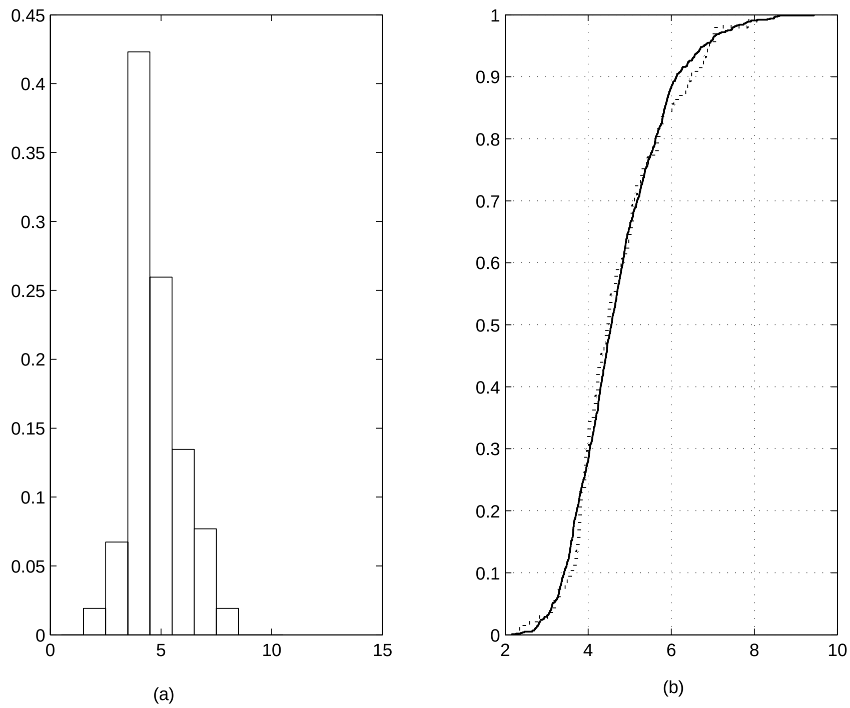


Figure 12. Log-transformed Non-Zero Catastrophe Losses. (a) Histogram; (b) Estimated (solid) and Empirical (dotted) Cumulative Distribution Function.

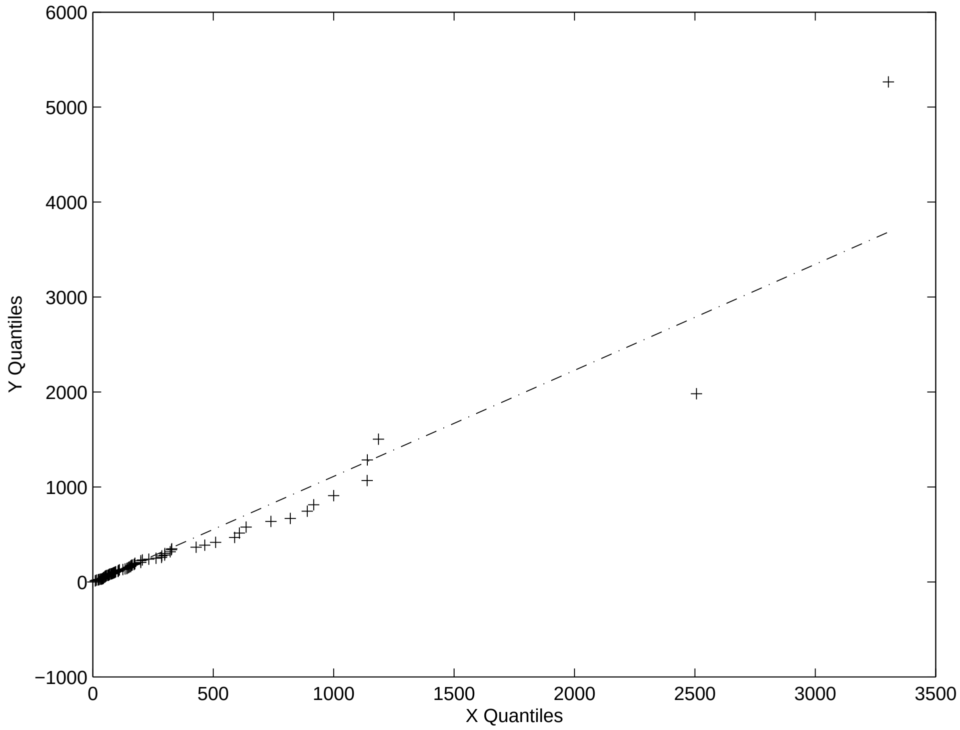


Figure 13. QQ Plot of Catastrophe Losses. Actual data versus simulated.

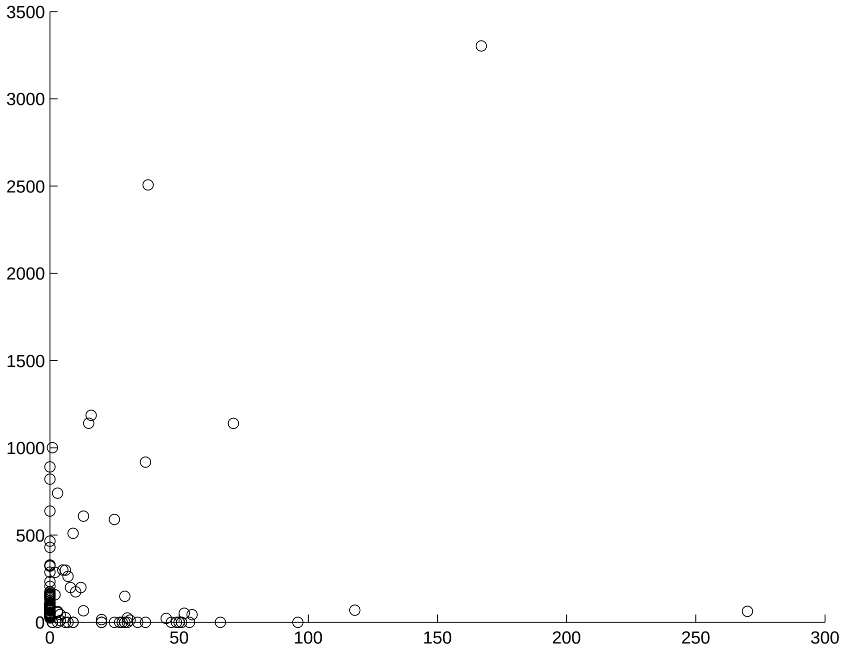


Figure 14. A Scatter Plot of Catastrophe Property Losses and Deaths.

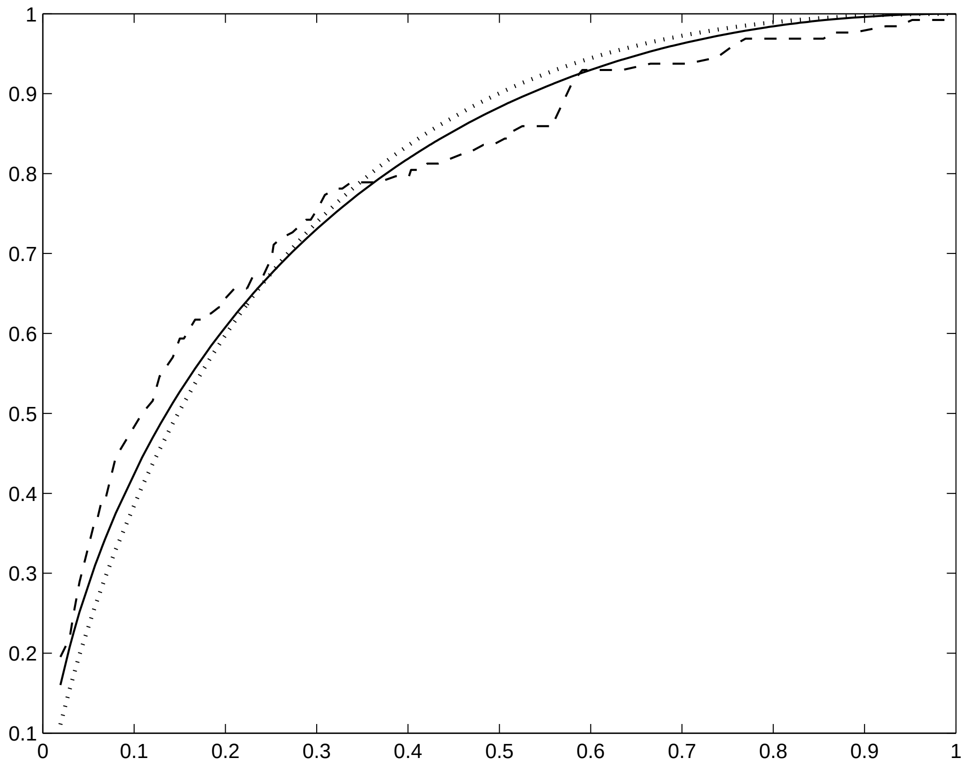


Figure 15. Copula Fit. $K_N(v)$ empirical (dashed) versus $K_{Clayton}(v)$ (solid) and $K_{Frank}(v)$ (dotted)

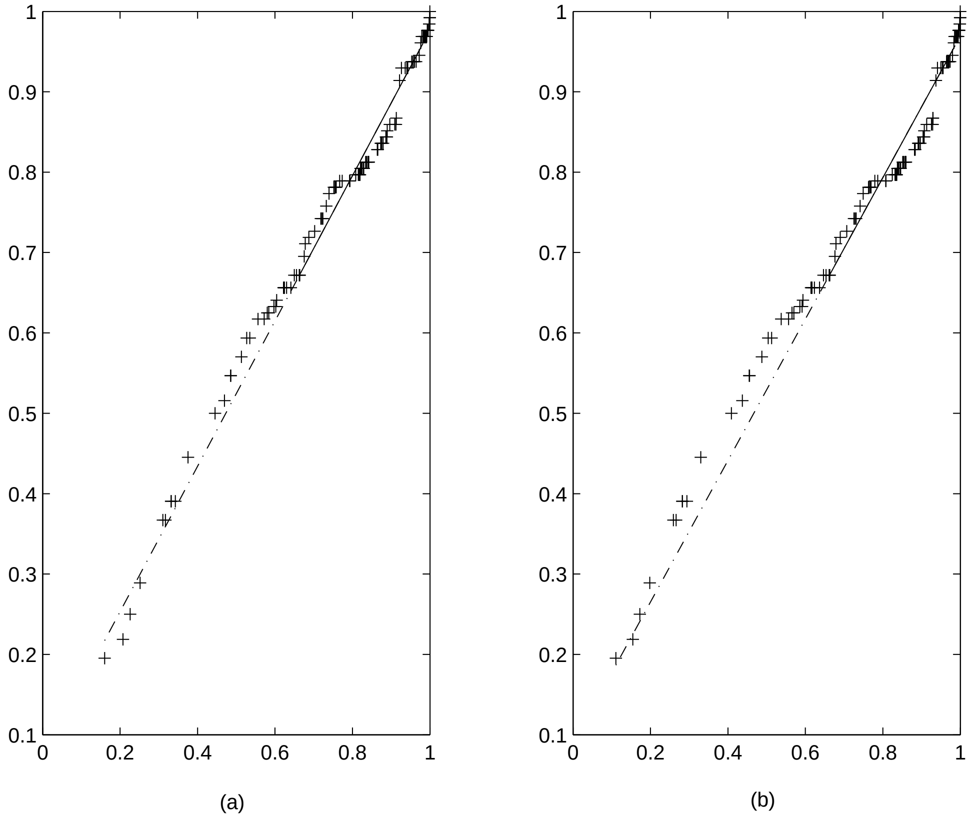


Figure 16. QQ-plot of Copula Fit. Empirical Copula versus (a) the Clayton and (b) the Frank copulas.

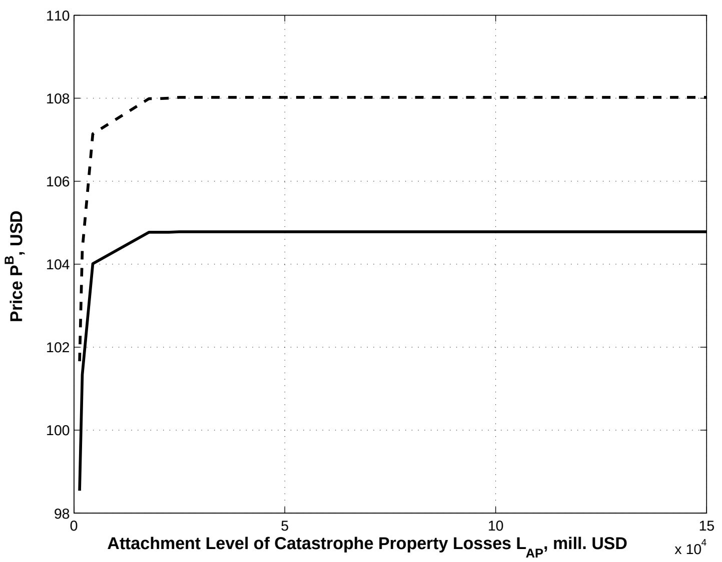


Figure 17. Price of the Coupon Paying CAT bond. Attachment level of deaths I_{AP}^{D} is 3000 (dash line) and 48 (solid line).

Appendix B: Tables

Date	Total	Natural Catastrophes	Man-Made Disasters	Terrorism Events
1970	1	0	1	0
1971	3	0	3	0
1972	2	0	2	0
1973	3	0	3	0
1974	1	0	1	0
1975	3	0	3	0
1976	0	0	0	0
1977	1	0	1	0
1978	0	0	0	0
1979	0	0	0	0
1980	3	0	3	0
1981	3	1	2	0
1982	1	1	0	0
1983	2	0	2	0
1984	4	0	4	0
1985	2	0	2	0
1986	1	0	1	0
1987	5	1	4	0
1988	6	0	6	1
1989	4	0	4	0
1990	6	4	2	0
1991	4	1	3	0
1992	3	0	2	2
1993	8	2	6	1
1994	11	3	8	0
1995	2	1	1	0
1996	6	1	5	2
1997	8	3	5	0
1998	8	4	4	1
1999	6	0	6	1
2000	4	1	3	0
2001	5	0	5	0
2002	6	3	3	0
2003	1	0	1	0
2004	3	1	2	0
2005	11	3	9	1
Total	137	30	107	9

Source: Swiss Re

Table 1. Number of Catastrophe Events, 1970-2005

	Case I	Time to Maturity Case II	Case III	Dependence Case IV	Case V	Threshold Level Case VI	Case VII
I_{AP}^D	48	48	48	48	48	**211**	**211**
L_{AP}	1.36	1.36	1.36	1.36	1.36	**4.34**	**4.34**
Kendall's τ	-0.16	-0.16	-0.16	**0.80**	**0.00**	-0.16	-0.16
Principal, F	100	100	100	100	100	100	100
Maturity, T	3	**2**	**4**	3	3	3	**4**
P^B	98.57	99.92	95.84	97.07	97.94	107.03	108.90
P_{ZC}^B	83.12	88.02	77.86	81.02	82.79	83.96	79.21

Table 2. Dynamics of the CAT Bond Price

L_{AP} is given in billions USD, F is 100 USD. P^B stands for a price of coupon-paying CAT bond and P_{ZC}^B for a price of a zero-coupon CAT bond. Prices are computed based on 1000 simulations.

	Catastrophe Event A			Catastrophe Event B		
I_{AP}^D	**3000**	**3000**	**3000**	**10000**	**10000**	**10000**
L_{AP}	**35.60**	**35.60**	**35.60**	**150.00**	**150.00**	**150.00**
Kendall's τ	-0.16	**0.80**	**0.80**	-0.16	**0.80**	**0.80**
Principal, F	100	100	100	100	100	100
Maturity, T	3	3	**4**	3	3	**4**
P^B	108.02	108.02	110.40	108.02	108.02	110.48
P_{ZC}^B	83.96	83.96	79.21	83.96	83.96	79.21

Table 3. High Magnitude Events

L_{AP} is given in billions USD, F is 100 USD. P^B stands for a price of coupon-paying CAT bond and P_{ZC}^B for a price of a zero-coupon CAT bond. Prices are computed based on 1000 simulations.

I_{AP}^{D}	48	48	48	48	48	48	48	48	48	48
L_{AP}	1.36	2.00	4.50	17.80	20.00	22.50	**25.00**	30.00	35.60	150.00
Kendall's τ	-0.16	-0.16	-0.16	-0.16	-0.16	-0.16	-0.16	-0.16	-0.16	-0.16
Principal, F	100	100	100	100	100	100	100	100	100	100
Maturity, T	3	3	3	3	3	3	3	3	3	3
P^{B}	98.57	101.34	104.01	104.77	104.77	104.77	**104.78**	104.78	104.78	104.78
P_{ZC}^{B}	83.12	83.54	83.88	**83.96**	83.96	83.96	83.96	83.96	83.96	83.96

Table 4. Price of the CAT bond with Attachment Level of Deaths of 48 people (80% quantile)

L_{AP} is given in billions USD, F is 100 USD. P^{B} stands for a price of coupon-paying CAT bond and P_{ZC}^{B} for a price of a zero-coupon CAT bond. Prices are computed based on 1000 simulations.

I_{AP}^{D}	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000
L_{AP}	1.36	2.00	4.50	17.80	20.00	22.50	**25.00**	30.00	35.60	150.00
Kendall's τ	-0.16	-0.16	-0.16	-0.16	-0.16	-0.16	-0.16	-0.16	-0.16	-0.16
Principal, F	100	100	100	100	100	100	100	100	100	100
Maturity, T	3	3	3	3	3	3	3	3	3	3
P^{B}	101.66	104.35	107.14	107.99	107.99	108	**108.02**	108.02	108.02	108.02
P_{ZC}^{B}	83.96	83.96	83.96	83.96	83.96	83.96	83.96	83.96	83.96	83.96

Table 5. Price of the CAT bond with Attachment Level of Deaths of 3000 people (the 9/11 Attacks)

L_{AP} is given in billions USD, F is 100 USD. P^{B} stands for a price of coupon-paying CAT bond and P_{ZC}^{B} for a price of a zero-coupon CAT bond. Prices are computed based on 1000 simulations.

Appendix C: One-Period Interest Rate

According to the representative agent model, viewed from time n, the price $P_n(CF)$ of a generic future cash flow process $CF = \{CF(k) \mid k = n+1, n+2, \ldots, T\}$ is given by a conditional expectation

$$P_n(CF) = E_{\mathbb{P}}\left[\sum_{k=n+1}^{T} \frac{u'_k(C^*(\omega,k))}{u'_n(C^*(\omega,n))} CF(k) | \mathscr{P}_n\right], \tag{3.9}$$

where $u_k(C^*(\omega,k))$ is the utility of the amount of the consumption good endowed to the entire economy in state ω at time k.

Consider time k. If there is available a cash flow stream that pays a unit in one period from now with certainty and nothing else, using equation (3.9), the implied interest rate $r(k)$ over the period is equal to

$$r(k) = \left[E_{\mathbb{P}}\left[\frac{u'_{k+1}(C^*(\omega,k+1))}{u'_k(C^*(\omega,k))} | \mathscr{P}_k\right]\right]^{-1} - 1, \tag{3.10}$$

leading to equation (3.2).

Appendix D: Valuation Framework

The probability space for the full model is given as $(\Omega, \mathscr{P}, \mathbb{P})$, where $\Omega := \Omega^{(1)} \times \Omega^{(2)}$; $\mathscr{P}_k = \mathscr{P}_k^{(1)} \times \mathscr{P}_k^{(2)}$ for $k = 0, 1, \ldots, T$; $\mathbb{P}(\omega) = \mathbb{P}_1(\omega^{(1)})\mathbb{P}_2(\omega^{(2)})$, where $\omega = (\omega^{(1)}, \omega^{(2)})$ is a generic state of the world describing the state of the financial market variables and the catastrophic risk variables. The financial market variables are modelled on the filtered probability space $(\Omega^{(1)}, \mathscr{P}^{(1)}, \mathbb{P}_1)$. The catastrophic risk variables are modelled on the filtered probability space $(\Omega^{(2)}, \mathscr{P}^{(2)}, \mathbb{P}_2)$. *In this setting, events that depend only on economic risk variables and those that depend only on catastrophic risk variables are independent.*

To explain formula (3.5) in the section 4.2 we show main steps of the proof developed in the paper by Cox and Pedersen (2000).

Authors define two new filtrations: $\mathscr{A}_k^{(1)} := \mathscr{P}_k^{(1)} \times \{\emptyset, \Omega^{(2)}\}$ for $k = 0, 1, \ldots, T$ and $\mathscr{A}_k^{(2)} := \{\emptyset, \Omega^{(1)}\} \times \mathscr{P}_k^{(2)}$ for $k = 0, 1, \ldots, T$. They prove that the two sigma-algebras $\mathscr{A}_T^{(1)}$ and $\mathscr{A}_T^{(2)}$ are *independent* under the probability measure $\mathscr{P}$:

$$\mathbb{P}(\alpha_1 \cap \alpha_2) = \mathbb{P}(\alpha_1)\mathbb{P}(\alpha_2),$$

where $\alpha_1 \in \mathscr{A}_T^{(1)}$, $\alpha_2 \in \mathscr{A}_T^{(2)}$ and $\alpha_1 = A_1 \times \Omega^{(2)}$ for some $A_1 \in \mathscr{P}_T^{(1)}$ and $\alpha_2 = \Omega^{(1)} \times A_2$ for some $A_2 \in \mathscr{P}_T^{(2)}$.

A random variable X on $(\Omega, \mathscr{P}, \mathbb{P})$ is said to depend only on financial risk variables if it is measurable with respect to $\mathscr{A}_T^{(1)}$. Alternatively, a random variable X is said to depend only on catastrophic risk variables if it is measurable with respect to $\mathscr{A}_T^{(2)}$.

A stochastic process Y is said to evolve through dependence only on financial risk variables if Y is adapted to $\mathscr{A}^{(1)}$. Alternatively, a stochastic process Y is said to evolve through dependence only on catastrophic risk variables if Y is adapted to $\mathscr{A}^{(2)}$.

In this setting and under the *assumption that the aggregate consumption depends only on the financial risk variables* implies that $C^*(\omega^{(1)}, \omega^{(2)}, k)$ is adapted to the filtration $\mathscr{A}^{(1)}$.

Lemma (p. 69-70, Cox and Pedersen (2000))

Under the assumption that aggregate consumption depends only on the financial risk variables, for any random variable X that depends only on catastrophic risk variables

$$E_{\mathbb{Q}}[X] = E_{\mathbb{P}}[X]. \tag{3.11}$$

In particular, for any catastrophe event $\alpha \in \mathscr{A}_T^{(2)}$ we have $\mathbb{Q}(\alpha) = \mathbb{P}(\alpha) = \mathbb{P}_2(A)$, where $\alpha \equiv \Omega^{(1)} \times A$ for some set $A \in \mathscr{P}_T^{(2)}$.

Proof

We see from equation (3.4) that the Radon-Nikodym derivative depends only on financial risk variables, that is $\frac{d\mathbb{Q}}{d\mathbb{P}} \mid_{\mathscr{P}_T}$ is $\mathscr{A}_T^{(1)}$ measurable. Therefore, the random variables X and $\frac{d\mathbb{Q}}{d\mathbb{P}} \mid_{\mathscr{P}_T}$ are independent under $\mathbb{P}$.

$$E_{\mathbb{Q}}[X] = E_{\mathbb{P}}[X \tfrac{d\mathbb{Q}}{d\mathbb{P}} \mid_{\mathscr{P}_T}] = E_{\mathbb{P}}[X] E_{\mathbb{P}}\left[\tfrac{d\mathbb{Q}}{d\mathbb{P}} \mid_{\mathscr{P}_T}\right] = E_{\mathbb{P}}[X] \cdot 1 = E_{\mathbb{P}}[X].$$

Let $X = 1_\alpha$ then

$\mathbb{Q}(\alpha) = E_{\mathbb{Q}}[1_\alpha] = E_{\mathbb{P}}[1_\alpha] = \mathbb{P}(\alpha) = \mathbb{P}(\Omega^{(1)} \times A) = \mathbb{P}_1(\Omega^{(1)})\mathbb{P}_2(A) = 1 \cdot \mathbb{P}_2(A) = \mathbb{P}_2(A).$

$\square$

Lemma (p. 70, Cox and Pedersen (2000))

Under the assumption that aggregate consumption depends only on the financial risk variables, the sigma-algebras $\mathscr{A}_T^{(1)}$ and $\mathscr{A}_T^{(2)}$ are independent under $\mathbb{Q}$.

Proof

Consider $\alpha_1 \in \mathscr{A}_T^{(1)}$ and $\alpha_2 \in \mathscr{A}_T^{(2)}$. $\mathbb{Q}(\alpha_1 \cap \alpha_2) = E_{\mathbb{Q}}\left[1_{\alpha_1} 1_{\alpha_2}\right] = E_{\mathbb{P}}\left[1_{\alpha_1} 1_{\alpha_2} \frac{d\mathbb{Q}}{d\mathbb{P}} \mid_{\mathscr{P}_T}\right]$.

Variables 1_{α_1}, 1_{α_2} and $\frac{d\mathbb{Q}}{d\mathbb{P}} \mid_{\mathscr{P}_T}$ are independent under $\mathbb{P}$ and therefore

$$E_{\mathbb{P}}\left[1_{\alpha_1} 1_{\alpha_2} \tfrac{d\mathbb{Q}}{d\mathbb{P}} \mid_{\mathscr{P}_T}\right] = E_{\mathbb{P}}\left[1_{\alpha_1} \tfrac{d\mathbb{Q}}{d\mathbb{P}} \mid_{\mathscr{P}_T}\right] E\left[1_{\alpha_2}\right] = \mathbb{Q}(\alpha_1)\mathbb{P}(\alpha_2) = \mathbb{Q}(\alpha_1)\mathbb{Q}(\alpha_2). \ \square$$

This result brings us back to the statement in the section 4.2 namely that under the valuation measure $\mathbb{Q}$ those events that depend only on financial risk variables are independent of those events that depend only on catastrophic risk variables.

If results of the above-mentioned lemmas are applied to formula (3.3) then the result in formula (3.5) is obtained. Note that cash flows $CF(k)$ on the bond are assumed to depend only on the catastrophic risk valuables and that aggregate consumption $C^*(\omega, k)$ is assumed to depend only on the financial risk variables.

To conclude, the main steps to arrive from formula (3.1) to (3.5) are the following

$$\begin{aligned}
P_0(CF) =& E_{\mathbb{P}}\left[\sum_{k=1}^{T} \frac{u_k'(C^*(\omega, k))}{u_0'(C^*(0))} CF(k)\right] \\
&= E_{\mathbb{Q}}\left[\sum_{k=1}^{T} \frac{1}{[1+r(0)][1+r(1)]\dots[1+r(k-1)]} CF(k)\right] \\
&= \sum_{k=1}^{T} E_{\mathbb{Q}}\left[\frac{1}{[1+r(0)][1+r(1)]\dots[1+r(k-1)]} CF(k)\right] \\
&= \sum_{k=1}^{T} E_{\mathbb{Q}}\left[\frac{1}{[1+r(0)][1+r(1)]\dots[1+r(k-1)]}\right] E_{\mathbb{Q}}[CF(k)] \\
&= \sum_{k=1}^{T} P(0,k) E_{\mathbb{Q}}[CF(k)] = \sum_{k=1}^{T} P(0,k) E_{\mathbb{P}}[CF(k)] = \sum_{k=1}^{T} P(0,k) E_{\mathbb{P}_2}[CF(k)].
\end{aligned}$$

Appendix E: Multiple-Event CAT Bond Price

$$
\begin{aligned}
\mathrm{P}_0^B(CF) &= \sum_{t\in\{1,\ldots,T\}} \mathrm{C}P(0,t)E_{\mathbb{P}_2}\left[\mathbb{I}_{\tau_3>t}\right] + \mathrm{F}P(0,T)E_{\mathbb{P}_2}\left[\mathbb{I}_{\tau_4>T}\right] \\
&= \sum_{t\in\{1,\ldots,T\}} \mathrm{C}P(0,t)\mathbb{P}_2[\tau_3>t] + \mathrm{F}P(0,T)\mathbb{P}_2[\tau_4>T] \\
&= \sum_{t\in\{1,\ldots,T\}} \mathrm{C}P(0,t)\mathbb{P}_2[\min\{\tau_1,\tau_2\}>t] + \mathrm{F}P(0,T)\mathbb{P}_2[\max\{\tau_1,\tau_2\}>T] \\
&= \sum_{t\in\{1,\ldots,T\}} \mathrm{C}P(0,t)\mathbb{P}_2[\tau_1>t,\tau_2>t] + \mathrm{F}P(0,T)[1-\mathbb{P}_2(\max(\tau_1,\tau_2)\le T)] \\
&= \sum_{t\in\{1,\ldots,T\}} \mathrm{C}P(0,t)\mathbb{P}_2[\tau_1>t,\tau_2>t] + \mathrm{F}P(0,T)[1-\mathbb{P}_2(\tau_1\le\tau_2\le T)-\mathbb{P}_2(\tau_2\le\tau_1\le T)] \\
&= \sum_{t\in\{1,\ldots,T\}} \mathrm{C}P(0,t)\mathbb{P}_2[I_t^{AL}\le I_{AP_t}^L, \sup_{\{0\le j\le t\}} I_j^D \le I_{AP}^D] + \mathrm{F}P(0,T)[1-\mathbb{P}_2(\tau_1\le\tau_2\le T)-\mathbb{P}_2(\tau_2\le\tau_1\le T)] \\
&= \sum_{t\in\{1,\ldots,T\}} \mathrm{C}P(0,t)\mathbb{P}_2\left[\sum_{i=0}^{t} I_i^L \le I_{AP_t}^L, \sup_{\{0\le j\le t\}} I_j^D \le I_{AP}^D\right] + \\
&\quad \mathrm{F}P(0,T)[1-\mathbb{P}_2(\tau_1\le\tau_2 \mid \tau_2\le T)\mathbb{P}_2(\tau_2\le T)-\mathbb{P}_2(\tau_2\le\tau_1 \mid \tau_1\le T)\mathbb{P}_2(\tau_1\le T)],
\end{aligned}
$$

where $\mathbb{P}_2$ is a physical probability measure governing catastrophic events.

Limitations and Possible Extensions

This thesis focuses on issues of risk management in relation to operational risk with a specific emphasis on terrorism risk. The first paper of the thesis introduces the issue of dependence between operational losses and how it can be accounted for in the value of capital charge for operational risk. The analysis is implemented on the simulated data sample since data on real operational losses are unavailable. The numerical evaluation implemented in the paper would provide a more precise picture as to capital charge and the dependence of operational losses if analysis was based on real operational loss data. Prompted by Basel II, banks have only recently started gathering these data and, to our knowledge, there is no publicly available source on operational losses. Calibration of the simulated data is done by relying on the results of empirical studies that use some industry data. However, these data usually cover only one year (reporting bias) and its quality (data inconsistencies) and, therefore, reliability is not ideal. Currently and for the near future, the lack and public inaccessibility of operational loss data is a major obstacle to study and to develop analytical approaches for operational risk. A possible extension of this paper would be to use the suggested methodology when more classes of risk are included in CaR computations[22].

The second paper of the thesis is an empirical study of the impact of a particular type of operational risk event, namely, terrorist attacks, on the behavior of stock, bond and commodity markets. Further research on the impact of terrorist events in the framework of suggested methodologies can be extended to some countries in the islamic world as well as to such countries as India and Israel. In addition, a diversification effect can be further investigated by looking at different portfolios of stocks, bonds and commodities.

[22]We implement numerical evaluation of the CaR when operational losses of two classes of risk are considered. According to the regulatory framework, operational losses of a bank are grouped in 56 classes.

Finally, the third paper suggests a model for pricing of a multiple-event coupon paying CAT bond. The bond that we consider covers exposure to catastrophic risk such as natural and man-made disasters, including terrorist events. Pricing of the bond is done in the incomplete market setting, however, under simplifying assumption of independence of events that are driven by only economic risk variables and those that depend only on catastrophic risk variables. In the real world, catastrophic events may lead to economic events. For example, the 9/11 attacks resulted in both catastrophic property losses and reduction by the Federal Reserve of interest rates. Here, the price of the bond would be affected by both the catastrophic element and economic element. Therefore, a natural extension of the current paper would be to develop a pricing framework when the above-mentioned assumption of independence is relaxed, i.e the assumption of dependence is introduced. Another possible extension of the work may lie in the area of modelling of pricing of the bond based on the catastrophic data for countries other than the UK.

Summary and Final Remarks

Increasing magnitude of operational losses over the last decade and their negative effect on financial industry have necessitated increasing attention to operational risk. This thesis covers such issues as loss dependence and quantification of operational risk exposure and empirical analysis of the impact of terrorist attacks on the behavior of financial markets. Finally, we propose a model for pricing of a multiple-event coupon paying CAT bond. This bond covers exposure to catastrophic risks such as natural and man-made disasters, including terrorist attacks.

Research on operational risk both by academics and practitioners is currently growing with a particular focus on quantitative methodologies and their implementation. In relation to terrorism risk and, in particular, the effect of this risk on financial markets, most of the research is of descriptive nature and focuses on the impact of very few terrorist events (often only those which occurred on September 11, 2001). Finally, existing literature on pricing of CAT bonds focuses mostly on models for standard CAT bonds that do not have a multiple-event structure and do not cover exposure to terrorism risk.

Our work consists of three papers contributes to existing research in several ways. The first paper addresses the issue of dependence between operational losses and how it can be accounted for in the value of capital at risk associated with operational risk. In contrast to the Loss Distribution Approach described by regulators in Basel II, this model accounts for the underlying dependence between aggregate losses of different classes of risk. Advancing previous research, the measurement of the above mentioned dependence accounts for *both*, dependence between loss frequencies and dependence between loss severities. By accounting for these two sources of dependence, our approach helps to decrease inaccuracy in the measurement of bank's exposure to operational risk. In addition, we demonstrate results of the numerical evaluation when instead of the VaR, a coherent and more conservative risk measure is used to compute a capital charge, namely the expected shortfall.

The second paper is empirical and one of the very few that study the link between terrorism and behavior of stock, bond and commodity markets. In contrast to impact studies that often employ only event-study methodology, in our paper we investigate the impact of terrorism using other methods as well. These are non-parametric methodology and a filtered GARCH with the Extreme Value Theory approach. Both methods are standard econometric tools. However, the way we apply them in this work is original.

The results of empirical analysis suggest several diversification strategies for investors who may be concerned about possible adverse effects of terrorism risk on their portfolios. When dealing with terrorism risk, investors should consider holding two groups of assets: those that are likely to react positively to terrorist attacks or those that have little or no negative sensitivity to this risk. In the first case, a U.S. Government bond index is the safest choice followed by such industry stocks as aero/defense and pharma/biotech. However, given that these stock markets may also exhibit a negative response, investing in these industries as a diversification strategy against terrorist attacks may not always work. In the second case, banking stock index may be good for investment. Note that, though this stock index is least sensitive to terrorist attacks, it exhibits significant negative return movements associated with financial crashes. In relation to terrorist attacks, investing in a composite commodity index is preferable to investing in gold only. in addition, another possible way to reduce negative exposure to terrorist events would be to avoid investing in insurance, travel and airline industry stocks. Finally, the response of financial markets to terrorist events suggests several strategies of trading derivatives. For example, investors can hold long positions in put options on the industry stocks that may react negatively to terrorist events (for example, airline and insurance industry stocks). Or, alternatively, they can invest in call options, where the underlying asset is a U.S. Government bond index.

The third paper is the first of its kind to develop a model for pricing of multiple-event coupon paying CAT bonds covering exposure to catastrophic risks, including terrorism risk. The pricing of the bond is implemented under the assumption of market incompleteness by relying on the representative agent pricing model. In contrast to the existing literature on pricing of standard CAT bonds, the payoffs on the bond in our model are linked to two types of underlying processes: catastrophic property losses and catastrophic mortality. Along with natural catastrophes and man-made disasters, our model views terrorism risk as one of the main exposure risks that may affect the cash flows of the bond. An important contribution of the third paper is that it provides a numerical evaluation of the price of the bond under consideration. We implement a Monte Carlo simulation of the price using the UK catastrophe data provided by Swiss Re. We consider different cases/scenarious for which we compute corresponding prices of coupon-bearing and zero-coupon CAT bonds.

The results of the third paper indicate that the price of the bond increases with threshold levels and decreases with stronger positive dependence between property losses and deaths. With respect to time to maturity, there is an inverse relationship between the price of the bond and its time to expiration. Although this relationship always works for a zero-coupon catastrophe bond, it may not always hold when the bond pays coupons. We find the bond's price to be less responsive to changes in dependence between property losses and deaths than to changes in the bond's time to maturity. We see that for a coupon paying CAT bond the impact of increasing attachment levels on the bond's price dominates the effect of longer maturity periods. The opposite relationship is observed for a zero-coupon CAT bond. The model suggested in this paper may be interesting to insurance and reinsurance companies and other financial institutions. These organizations may want to consider using multiple-event CAT bonds to transfer to capital markets their exposure to catastrophic risks, including terrorism risk.

In summary, in this thesis we look at different aspects of operational risk management with a special emphasis on terrorism risk. As evidenced by the 9/11 attacks, terrorism risk can be catastrophic and can have a severe negative economic impact on financial industry. Quantification and management of terrorism risk represents an area of opportunity for further academic research. However, the high level of uncertainty surrounding terrorism risk makes this a very challenging task.

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